

EBA1180 Mathematics for Data Science
autumn 2023
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lectures 2 and 3

Sec. 2.2, 11.1, 11.3: Rate of change. Powers and roots. Interest. Present value of cash flow.

Here are recommended exercises from the textbook [SHSC].

Section 2.2 exercise 1-14

Section 11.1 exercise 1-9

Section 11.3 exercise 1a, 2a

Problems for the exercise session
Wednesday 30 Aug. at 12-17+ in B2-040

Problem 1

- a) The ticket price Oslo-Bergen is changed from 699 to 899 NOK. Determine the relative change and the growth factor.
- b) The ticket price Oslo-Bergen is changed from 899 to 699 NOK. Determine the relative change and the growth factor.
- c) Determine the product of the growth factors in (a) and (b). Interpret the answer as a growth factor.
- d) Suppose a and b are two numbers different from 0. Show that

$$\left(1 + \frac{b-a}{a}\right) \cdot \left(1 + \frac{a-b}{b}\right) = 1$$

and use this to explain the result in (c).

Problem 2 You deposit 50 000 in an account with 3.6% nominal annual interest.

- a) Suppose interest is added annually.
 - i) Calculate the balance after 10 years.
 - ii) Determine the growth factor and the relative change for these 10 years.
- b) Suppose interest is added monthly.
 - i) Calculate the balance after 10 years.
 - ii) Determine the growth factor and the relative change for these 10 years.
 - iii) Determine the effective annual interest.

Problem 3 (Christmas exam resit spring 2017, Problem 1) A bank account has a nominal annual interest 6.40% with interest paid monthly. What is the effective interest?

- (A) 6.40%
- (B) Between 6.40% and 6.50%
- (C) Between 6.50% and 6.60%
- (D) More than 6.60%
- (E) I choose not to answer this question.

Problem 4

- a) Determine what amount you have to deposit today to have 250 000 after 10 years if the interest is 3.4%.
- b) After 4 years (with the deposit and interest in (a)) the interest is changed to 1.9%. Determine the balance after 10 years.
- c) Explain why the answer in (b) is given by the expression $250\,000 \cdot \left(\frac{1.019}{1.034}\right)^6$.
- d) Determine the amount you would have to deposit in the case of (b) for the balance to be 250 000 after 10 years.
- e) Explain why the answer in (d) is given by the expression $\frac{250\,000}{1.019^6 \cdot 1.034^4}$.

Problem 5

- a) Determine the present discounted value of 70 million paid 4 years from now when the interest is 9%.

Suppose 70 million is paid after 5 years. Let r be the interest which gives the same present discounted value as in (a).

- b) Do you think that r is bigger or smaller than 9%? – find arguments, talk with someone!
- c) Determine r .
- d) Explain why the answer in (c) can be written as $1.09^{\frac{4}{5}} - 1$.

Problem 6 A payment of 20 million today is supposed to give a return of 9 million 4 years from now and another 14 million 7 years from now.

- a) Suppose the interest is 12%. Determine the present value of the investment.
- b) Do you think the interest has to increase or decrease for the investment to have 0 present value? – find arguments, talk with someone!
- c) Show that the internal rate of return is 2.44%.
- d) Do you think this investment sounds interesting? Give arguments, talk with someone!

Problem 7 A pharmaceutical company plans to test a new drug and then sell the patent. The testing lasts for 5 years and costs 400 million each year (assumed to be paid at the end of the year). The patent is sold immediately after testing ends.

- a) Suppose the rate of discount is 12%. What should the patent cost for the rate of discount to equal the internal rate of return for the cash flow?
- b) The patent is sold for 3.20 billion. Determine the internal rate of return of the cash flow.

Problem 8 We have the cash flow

Year	0	3	5	7	8
Payment	-30	-15	4	11	48

Suppose the interest is 9%.

- a) Calculate the future value of the cash flow after 8 years.
- b) Calculate the present value.

Suppose the interest is 13%.

- c) Calculate the future value of the cash flow after 8 years.
- d) Calculate the present value.
- e) Suppose r is the interest, and that the future value of the cash flow after n years is K_n . Then K_0 in particular is the present value of the cash flow. Explain why

$$K_8 = K_0 \cdot (1 + r)^8$$

and check that this is correct in (a-b) and (c-d). Explain why the sign of K_0 is the same as the sign of K_8 .

Problem 9

Hege and Kåre are saving some of their salaries. Hege saves 5 300/month. Kåre saves 4 800/month. The yearly nominal interest is 3.6% and the interest is added each month. When they start the saving Kåre already has 200 000 in the account, a gift from his dad because he (broadly speaking) hasn't taken up smoking. Hege has nothing.

When is Hege overtaking Kåre?

Answers

Problem 1

- a) Relative change is $r_1 = \frac{899-699}{699} = 28.61\%$. Growth factor is $1 + r_1 = 1.2861$.
b) Relative change: $r_2 = -22.25\%$. Growth factor: 0.7775.
c) $(1 + r_1)(1 + r_2) = 1$.

Problem 2

- a) i) 71 214.36 ii) Growth factor: 1.4243. Relative change: 42.43%.
b) ii) 71 627.86 ii) Growth factor: 1.4326. Relative change: 43.26% iii) 3.66%.

Problem 3 6.591% (C)

Problem 4

- a) 178 951.20
b) 229 013.92
d) 195 349.70

Problem 5

- a) 49.58 million
c) 7.14%

Problem 6

- a) -7.95 million

Problem 7

- a) 2.541 billion
b) $r = 23.7\%$ (Hint: You should get the equation $(r + 1)^5 - 8r - 1 = 0$. Use e.g. GeoGebra or Wolfram Alpha to draw the graph and read off the root.)

Problem 8

- a) -17.69
b) -8.88
c) -41.19
d) -15.49

Problem 9 Hint: Kåre has 200 000 in his account. After working for one month Hege and Kåre are paid their salary. They make their deposits as described. Compare the difference of the new balances with the difference of the balances one month earlier.