

EBA1180 Mathematics for Data Science
autumn 2023
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lecture 1 – Basic algebra (Sec. 2.1-6, 2.8)

Here are recommended exercises from the textbook [SHSC].

- Section 2.1 exercise 1
- Section 2.2 exercise 1-9
- Section 2.3 exercise 1-7, 9-11
- Section 2.4 exercise 1-8
- Section 2.5 exercise 1, 3, 3-7, 9-11
- Section 2.7 exercise 1-4

Problems for the exercise session
Wednesday 23 Aug. at 12-17+ in D1-065

Problem 1 Simplify the fractions as much as possible.

- a) $\frac{36}{120}$ b) $\frac{60}{48}$ c) $\frac{96}{69}$ d) $\frac{112}{12}$ e) $\frac{64}{1024}$ f) $\frac{91}{26}$
g) $\frac{4}{5} \cdot \frac{2}{3}$ h) $\frac{4}{15} \cdot \frac{3}{7}$ i) $\frac{18}{4} \cdot \frac{2}{3}$ j) $\frac{2}{3}$ k) $\frac{2}{3}$ l) $\frac{2}{3}$

Problem 2 Simplify the fractions as much as possible.

- a) $\frac{x^2y^2}{xy^3z}$ b) $\frac{15a(b+6)}{9(b+6)c}$ c) $\frac{x(x+1)}{(x+1)(x+2)}$ d) $\frac{x(x^2+1)}{x^2(x+1)}$
e) $\frac{(x+y)^2}{3x+3y}$ f) $\frac{(x+y)^3}{x^2+2xy+y^2}$ g) $\frac{x(x+5)}{2x^2+10x}$ h) $\frac{x^2-5}{(x-\sqrt{5})(x+\sqrt{5})}$
i) $\frac{x^2-3x}{x(y-3)} \cdot \frac{xy^2-9x}{x-3}$ j) $\frac{(3x-y)^2+6xy}{9x^2+y^2}$

Problem 3 Write as one fraction in the simplest way possible.

- a) $\frac{2}{x} + \frac{3}{x}$ b) $\frac{2}{x} + \frac{3}{y}$ c) $\frac{x-4}{x} - \frac{x}{x-4}$ d) $x+3 + \frac{2}{x-1}$
e) $\frac{1}{\sqrt{7}-1} - \frac{1}{\sqrt{7}+1}$ f) $\frac{1}{x-1} - \frac{1}{x+1}$ g) $\frac{x+3}{x^2+1} - \frac{1}{x-3}$ h) $\frac{x(x+3)}{(x+1)(x+2)} - 1$

Problem 4 If you replace y with x in the answer to problem 3b you should get the answer in problem 3a. Why? Is it true?

What do you get if x is replaced by $\sqrt{7}$ in the answer to problem 3f? Compare with problem 3e.

Problem 5 Compute without using digital assistance.

a) 2^3 b) 3^2 c) -3^2 d) $(-3)^2$ e) -2^3 f) $(-2)^3$

Problem 6 Compute/simplify without using digital assistance.

a) $\frac{4^2}{2^6}$ b) $\frac{2^5 \cdot 10^3}{5^3}$ c) $\frac{3^{10}}{3^9}$ d) $\frac{3^9}{3^{10}}$ e) $\frac{35^8}{5^7 \cdot 7^8}$

f) $\left(\frac{1}{2}\right)^3$ g) $\left(\frac{2}{5}\right)^3$ h) $(\sqrt{7})^2$ i) $(\sqrt{7})^3$ j) $\frac{7^2}{\sqrt{7} \cdot 7}$

k) $\frac{6^2}{\sqrt{2} \cdot \sqrt{3}}$ l) $\frac{(\sqrt{1,03})^{10}}{1.03^4}$ m) $\sqrt{3^2 + 4^2}$ n) $(\sqrt[3]{5})^3$ o) $(\sqrt[4]{7})^8$

p) $\sqrt[6]{27}$ q) $|-3.2|$ r) $|4.3 - 5.9|$

Problem 7 Solve the equations.

a) $x^2 = 9$ b) $x^2 = -9$ c) $\sqrt{x} = 9$ d) $\sqrt{x} = -9$

e) $(x - 4)^2 = 9$ f) $(x + 7)^3 = 27$ g) $|x| = 25$ h) $|x| = -1$

i) $\frac{1}{|x|} = 0.25$ j) $x|x| = 9$ k) $|x - 2| = 25$ l) $|x - 3| = 3$

Problem 8 Solve the equations (a is an arbitrary number).

a) $x^2 = 3^2$ b) $x^2 = (-3)^2$ c) $x^2 = a^2$

d) $x^3 = 2^3$ e) $x^3 = (-2)^3$ f) $x^3 = a^3$

g) $x^2 = a^{-4}$ h) $x^3 = 1.03^{-12}$ i) $x^4 = a^{-4}$

Answers

Problem 1

- a) $\frac{3}{10}$ b) $\frac{5}{4}$ c) $\frac{32}{23}$ d) $\frac{28}{3}$ e) $\frac{1}{16}$ f) $\frac{7}{2}$
 g) $\frac{8}{15}$ h) $\frac{4}{35}$ i) $\frac{1}{4}$ j) $\frac{2}{9}$ k) 1 l) $\frac{4}{9}$

Problem 2

- a) $\frac{x}{yz}$ b) $\frac{5a}{3c}$ c) $\frac{x}{x+2}$ d) $\frac{x^2+1}{x(x+1)}$ e) $\frac{1}{3}(x+y)$
 f) $x+y$ g) $\frac{1}{2}$ h) 1 i) $x(y+3)$ j) 1

Problem 3

- a) $\frac{5}{x}$ b) $\frac{3x+2y}{xy}$ c) $\frac{8(2-x)}{x(x-4)}$ d) $\frac{x^2+2x-1}{x-1}$
 e) $\frac{1}{3}$ f) $\frac{2}{(x-1)(x+1)}$ g) $\frac{-10}{(x^2+1)(x-3)}$ h) $\frac{-2}{(x+1)(x+2)}$

Problem 5

- a) 8 b) 9 c) -9 d) 9 e) -8 f) -8

Problem 6

- a) $\frac{1}{4}$ b) 2^8 c) 3 d) $\frac{1}{3}$ e) 5 f) $\frac{1}{8}$
 g) $\frac{2^3}{5^3}$ h) 7 i) $7\sqrt{7}$ j) $\sqrt{7}$ k) $6\sqrt{6}$ l) 1,03
 m) 5 n) 5 o) 49 p) $\sqrt{3}$ q) 3,2 r) 1,6

Problem 7

- a) $x = 3, x = -3$ b) no solution c) $x = 81$ d) no solution
 e) $x = 1, x = 7$ f) $x = -4$ g) $x = 25, x = -25$ h) no solution
 i) $x = 4, x = -4$ j) $x = 3$ k) $x = -23, x = 27$ l) $x = 0, x = 6$

Problem 8

- a) $x = 3, x = -3$ b) $x = 3, x = -3$ c) $x = |a|, x = -|a|$, which is the same as $x = \pm a$
 d) $x = 2$ e) $x = -2$ f) $x = a$
 g) $x = a^{-2}, x = -a^{-2}$ h) $x = 0,888$ i) $x = \frac{1}{a}, x = -\frac{1}{a}$