EBA1180, lecture 9, 20 Sept. 2023, Runar Ile

Plan
1. Rational equations
2. Radical equations
3. Inequalities

1. Rational equations

A rational eq: $\dfrac{p(x)}{q(x)} = 0$

polynomials

Ex. Eq. $\dfrac{x+1}{(x-1)(x+3)} = 0$ then $x+1=0$

and $(x-1)(x+3) \neq 0$

i.e. $x \neq 1$, $x \neq -3$

so $\underline{\underline{x=-1}}$

Ex (Probl. 10a from last week). Solve the eq.

$$1 + x + x^2 + \dots + x^{99} = 0$$

Solution This is a geometric series with

$a_1 = 1$, $k = x$, number of terms $= 100$

The formula gives the LHS of the eq:

$$1 \cdot \frac{x^{100}-1}{x-1} = 0 \qquad (x \neq 1)$$

Then $x^{100} - 1 = 0$ so $x^{100} = 1$ $(x \neq 1)$

so $x = \pm 1^{\frac{1}{100}} = \pm 1$ $(x \neq 1)$

so $\underline{\underline{x = -1}}$ (Note also that LHS $= 100$ with $x = 1$)

$(*)$

Ex $$\frac{x+1}{(x-1)(x+3)} = 2 \qquad 1-2$$

$$\frac{x+1}{(x-1)(x+3)} - 2 = 0$$

Multiply -2 with $\frac{(x-1)(x+3)}{(x-1)(x+3)} = 1$

Get $$\frac{x+1 - 2(x-1)(x+3)}{(x-1)(x+3)} = 0$$

Resolve the parantheses

$$\frac{x+1-2(x^2+2x-3)}{(x-1)(x+3)} = 0$$

collect terms

$$\frac{-2x^2-3x+7}{(x-1)(x+3)} = 0$$

that is: $-2x^2-3x+7=0$

and $x \neq 1$, $x \neq -3$

which you can solve.

<u>Note</u>: could also multiply each side of $(*)$ with $(x-1)(x+3)$

(and remember that $x \neq 1$, $x \neq -3$).

2. Radical equations

- the unknown is under a root

Ex $2\sqrt{x+1} = x-2$ $\quad (x \geq -1)$

Square both sides.

$$4(x+1) = (x-2)^2 = (x-2)(x-2) = x^2-4x+4$$

$$4x+\cancel{4} = x^2-4x+\cancel{4}$$

$$x^2-8x = 0$$

$$x(x-8) = 0 \quad \text{so} \quad \underline{x=0} \quad \text{or} \quad \underline{x=8}$$

Note Not all of these x-values need to be solutions of the original eq. (think: $-3 \neq 3$ but $(-3)^2 = 3^2$).

We have to test the candidates:

$\underline{x=0}$ LHS: $2\cdot\sqrt{0+1} = 2\sqrt{1} = 2$
RHS: $0-2 = -2$
} not equal so $x=0$ is not a solution

$\underline{x=8}$ LHS: $2\cdot\sqrt{8+1} = 2\sqrt{9} = 6$
RHS: $8-2 = 6$
} - equal! so $\underline{\underline{x=8}}$ is the only solution.

3. Inequalities

$-2 < -1$ read: " minus two is less than minus one"

$\frac{1}{9} > \frac{1}{12}$ read: "one ninth is greater than one twelfth"

Also $\leq$ and $\geq$

Start 11.01

- An inequality is a claim that one expression (number) is less than (bigger than, ...) another expression (number).

- The solutions of an inequality are those values of x which make the claim true.

Ex $x - 1 \geq 2$ is a claim.

$*$ is true if $x = 5$ since $5 - 1 = 4 \geq 2$ is true

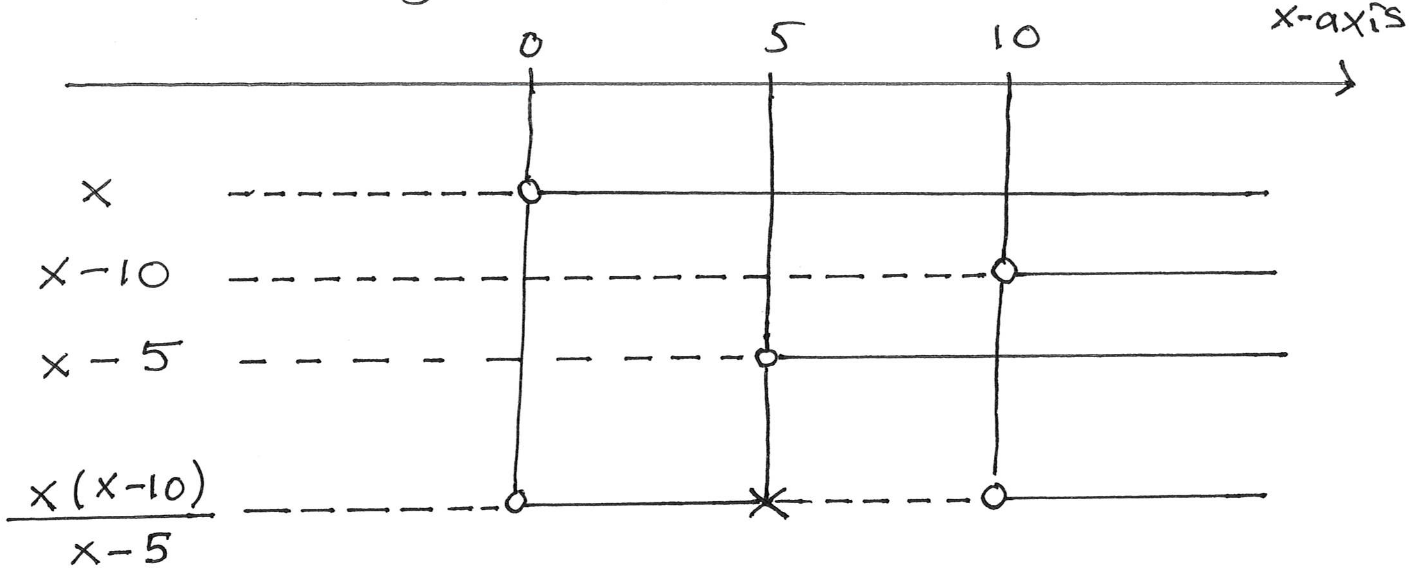
$*$ is not true if x 2 since $2 - 1 = 1 \geq 2$ is not true

The solutions of the inequality are the values of x such that

$$x \geq 3$$

Ex Solve the inequality $\dfrac{x(x-10)}{x-5} \geq 0$

Solution Because we have 0 on the RHS and factorised LHS we can use a sign diagram directly :



that is $0 \leq x < 5$ or $x \geq 10$

We also write $x \in [0, 5\rangle$ or $x \in [10, \infty\rangle$

Ex $$\frac{2x-12}{(x-3)(x+4)} \geq 1$$ 1-1 (Course Paper 2020a)

Solution Equivalent inequality :

$$\frac{2x-12}{(x-3)(x+4)} - 1 \cdot \frac{(x-3)(x+4)}{(x-3)(x+4)} \geq 0$$

that is $$\frac{2x-12-(x-3)(x+4)}{(x-3)(x+4)} \geq 0$$

resolve and collect in the numerator

$$\frac{2x - 12 - (x^2 + x - 12)}{(x-3)(x+4)} \geq 0$$

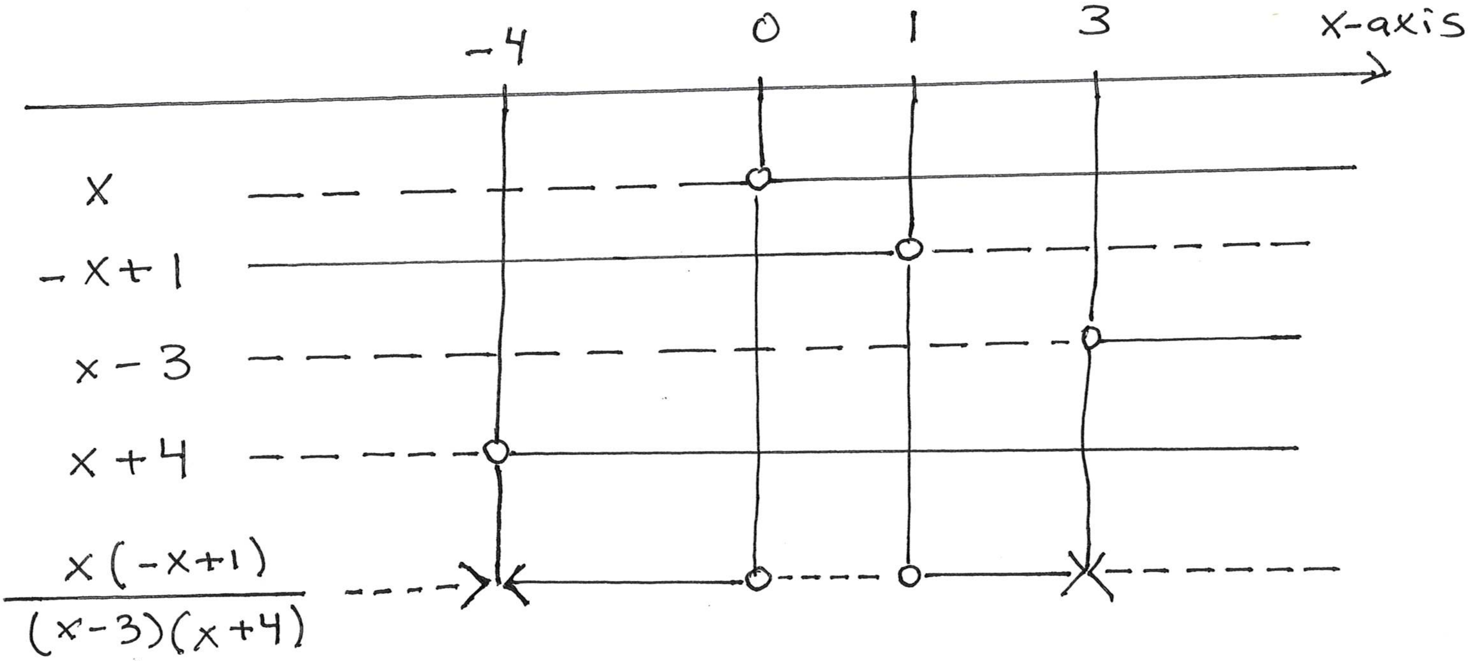
$$\frac{-x^2 + x}{(x-3)(x+4)} \geq 0$$

$$\frac{x(-x+1)}{(x-3)(x+4)} \geq 0$$

zero on the RHS

factorised LHS

– Ready for a sign diagram:



That is $\underline{\underline{-4 < x \leq 0 \text{ or } 1 \leq x < 3}}$

Alternate way of writing $\underline{\underline{x \in \langle -4, 0] \text{ or } x \in [1, 3\rangle}}$