EBA 1180, lecture 8, 19 Sept. 2023, Runar Ile

Plan 1. Repetition (problems from last week)
2. Polynomial division and factorisation

1. Repetition

2m) Solve the eq. $9x^2 - 6x + 1 = 0$ 1 : 9

$$x^2 - \frac{2}{3}x + \frac{1}{9} = 0$$

Complete the square: $(x - \frac{1}{3})^2 \underbrace{- (\frac{1}{3})^2 + \frac{1}{9}}_{=0} = 0$

so $(x - \frac{1}{3})^2 = 0$ so $x - \frac{1}{3} = 0$

so $\underline{\underline{x = \frac{1}{3}}}$

Alternative solution: Put $u = 3x$

so $u^2 = (3x)^2 = 3^2 \cdot x^2 = 9x^2$.

So the eq. becomes $u^2 - 2u + 1 = 0$

so $(u-1)^2 = 0$

so $u = 1$

so $3x = 1$

so $\underline{\underline{x = \frac{1}{3}}}$

3e) Determine the quadratic eq. with the given solutions:

$x = 3 \pm \sqrt{5}$, that is $x = 3 + \sqrt{5}$, $x = 3 - \sqrt{5}$

Then $(x - (3+\sqrt{5})) \cdot (x - (3 - \sqrt{5}))$

$= x^2 - (3-\sqrt{5})x - (3+\sqrt{5})x + (3+\sqrt{5})(3-\sqrt{5})$

$= x^2 - 6x + 4$ so $\underline{\underline{x^2 - 6x + 4 = 0}}$ has the given solutions.

5c) Determine k such that

$$\frac{1}{k}x^2 - 14x = 12$$ has exatly one solution.

Note: $k \neq 0$. Multiply BS with k:

$$x^2 - 14kx = 12k$$

Complete the sq:

$$(x - 7k)^2 = 12k + (7k)^2$$

has exacty one solution if and only if the RHS $= 0$, that is

$$12k + 49k^2 = 0$$

i.e. $$k(12 + 49k) = 0$$

i.e. $k = 0$ or $12 + 49k = 0$

– not allowed!

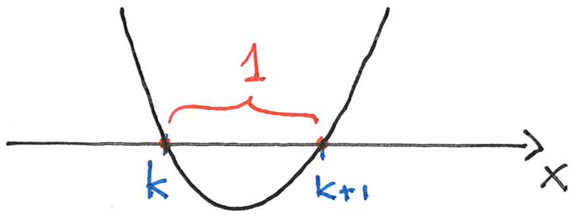
$$\underline{\underline{k = -\frac{12}{49}}}$$

<u>Parametres</u>: numbers without explicit values – used to describe many situations simultaneously.

(economists: "exogenous variables")

(the x: "endogenous variable").

Probl 7a All polynomicals x^2+bx+c which have <u>two</u> zeros of distance 1 from each other



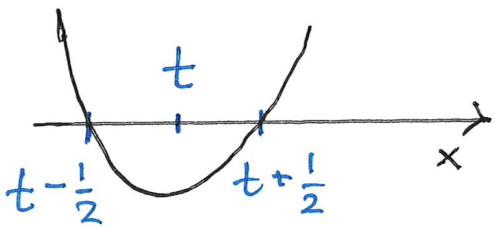
can be written as

$$\overbrace{(x-k)}^{\text{zero: } x=k}\cdot\big(\underbrace{x-(k+1)}_{\text{zero: } x=k+1}\big)$$

where k is the smallest zero (root)

Then $(x-k)(x-(k+1)) = \underline{\underline{x^2-(2k+1)x+k(k+1)}}$

<u>or</u>



Get

$$\left(x-\left(t-\tfrac{1}{2}\right)\right)\cdot\left(x-\left(t+\tfrac{1}{2}\right)\right)$$

$$= \underline{\underline{x^2-2tx+t^2-\tfrac{1}{4}}}$$

Infinitly many correct solutions.
(this was just two of them)

Start : 11.01

2. Polynomial division and factorisation

Want to divide a polynomial $f(x)$ with a polynomial $g(x)$ with a remainder $r(x)$.

$$g(x) \cdot \Big| \quad \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{with } \deg(r(x)) < \deg(g(x))$$

gives $f(x) = q(x) \cdot g(x) + r(x)$

Ex $f(x) = 3x^2 + 2x + 1$ and $g(x) = x - 2$

$$\underbrace{(\boxed{3x^2} + 2x + 1) : (\boxed{x} - 2)}_{} = \overset{3x^2 : x}{\overset{\shortparallel}{3x}} + \overset{8x : x}{\overset{\shortparallel}{8}} + \frac{17}{x-2}$$

$$\begin{array}{l} -(3x^2 - 6x) \quad \leftarrow \cdot (x-2) \\ \hline \qquad \boxed{8x} + 1 \\ \quad -(8x - 16) \quad \leftarrow \cdot (x-2) \\ \hline \qquad\quad 17 \end{array}$$

17 is called the remainder

So $\underline{\underline{q(x) = 3x + 8 \quad \text{and} \quad r(x) = 17}}$

Can check: $\left(3x + 8 + \frac{17}{x-2}\right) \cdot (x-2)$

$$= (3x+8)(x-2) + \frac{17}{x-2} \cdot (x-2)$$

$$= 3x^2 - 6x + 8x - 16 + 17 = 3x^2 + 2x + 1 = f(x)$$

– so ok!

Two applications of polynomial division

Ⓐ To find asymptotes of rational functions

Ex $\quad \dfrac{3x^2+2x+1}{x-2} = 3x+8 + \dfrac{17}{x-2}$

has a vertical asymptote: the line $x=2$

and a non-vertical asymptote: the line $y=3x+8$
("oblique")

Ⓑ To factorise a polynomial as a product of degree 1 (linear) polynomials.

Ex Factorise $x^3-4x^2-11x+30$ into linear factors.

Solution Three steps.

Step I Guess an integer root (zero).
[Note: Has to divide 30]

I try $x=-3$ and get (check):

$(-3)^3-4\cdot(-3)^2-11\cdot(-3)+30$

$=-27-36+33+30 = 0$

Then $(x-(-3)) = (x+3)$ is a factor.

Step II Use polynomial division to find a polynomial of lower degree:

$(x^3-4x^2-11x+30):(x+3) \overset{\text{by poly. division!}}{=} x^2-7x+10$

Note: Remainder is 0! poly. division..

Step III We find the roots of $x^2-7x+10$

They are $x=2$, $x=5$

so $x^2-7x+10 = (x-2)(x-5)$.

Then $x^3-4x^2-11x+30 = \underline{\underline{(x-2)(x-5)(x+3)}}$

Note 1 Not always possible to factorise !

Ex x^2+5 has no roots !

x^2+2x+3 —— " —— $(b^2-4ac = 2^2-4\cdot1\cdot3<0)$

Note 2 It can be difficult to guess roots
– and roots don't have to be integers.