EBA1180, lecture 7, 13 Sept. 2023, Runar Ile

Plan
1. Quadratic equations
2. Completing the square
3. Equations with given solutions

1. Quadratic equations

- an eq. which can be transformed into the standard form $ax^2 + bx + c = 0$ $(a \neq 0)$

It has solution(s):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Three cases:

$b^2 - 4ac > 0$ gives <u>two</u> solutions

$b^2 - 4ac = 0$ gives <u>one</u> solution

$b^2 - 4ac < 0$ gives <u>no</u> solution

<u>Problem</u> Determine the number of solutions.

a) $x^2 + 5x + 6 = 0$ $\quad 5^2 - 4 \cdot 1 \cdot 6 > 0$: two solutions

b) $-x^2 + 2x - 1 = 0$ $\quad 2^2 - 4 \cdot (-1) \cdot (-1) = 0$: one solution

c) $4x^2 - 5x - 5 = 0$ $\quad (-5)^2 - 4 \cdot 4 \cdot (-5) > 0$: two solutions

The quadratic (abc) - formula is often inefficient:

<u>EX</u> $-3x^2 + 7 = 0$ $\quad (a = -3,\ \underline{b = 0},\ c = 7)$

$$-3x^2 = -7 \quad | :(-3)$$

$$x^2 = \frac{7}{3}$$

$$|x| = \sqrt{x^2} = \sqrt{\frac{7}{3}} \quad \text{so} \quad \underline{\underline{x = \pm\sqrt{\frac{7}{3}}}}$$

Ex $2x^2 - 6x = 0$ $\quad (a=2,\ b=-6,\ c=0)$

$2(x^2 - 3x) = 0$ $\quad | : 2$

$x^2 - 3x = 0$

$x(x-3) = 0$ $\quad$ then

either $\underline{\underline{x = 0}}$ $\quad$ or $\quad x - 3 = 0$

$\underline{\underline{x = 3}}$

Pattern: If $a \cdot b = 0$ then $a = 0$ or $b = 0$ (or both)

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2. Completing the square

Ex $\underbrace{x^2 + 6x} - 16 = 0$

Claim: $x^2 + 6x = (x+3)^2 - 9$

-because $(x+3)^2 = x^2 + 2 \cdot 3x + 3^2$

$= x^2 + 6x + 9$

$6:2 \qquad 3^2$

$\underbrace{(x+3)^2 - 9} - 16 = 0$

$(x+3)^2 = 25$

so $\quad x + 3 = 5$ $\quad$ or $\quad x + 3 = -5$

$\underline{\underline{x = 2}}$ $\qquad \underline{\underline{x = -8}}$

Problem Solve the quadratic eq.s by completing the square

a) $\underbrace{x^2 - 8x} - 33 = 0$

Solution $\frac{-8}{2} = -4$ so $x^2 - 8x = (x-4)^2 - (-4)^2$

(because $(x-4)^2 = x^2 - 2\cdot 4x + (-4)^2 = x^2 - 8x + 16$

Rewrite eq: $\underbrace{(x-4)^2 - 16} - 33 = 0$

$(x-4)^2 = 49$

so $x - 4 = 7$ or $x - 4 = -7$

$\underline{\underline{x = 11}}$ $\underline{\underline{x = -3}}$

b) $\underbrace{x^2 + 2x} = 63$

Solution $x^2 + 2x = (x+1)^2 - 1^2$ so

rewrite eq: $\underbrace{(x+1)^2 - 1} = 63$

so $(x+1)^2 = 64$

so $x + 1 = 8$ or $x + 1 = -8$

$\underline{\underline{x = 7}}$ $\underline{\underline{x = -9}}$

Start: 11.03

3. Equations with given solutions

Problem Solve the equation

$$(x-4)(x+5) = 0$$

Solution If a product of two numbers is equal to zero: $a \cdot b = 0$ then at least one of the numbers has to be zero: $a=0$ or $b=0$

So when $(x-4)\cdot(x+5) = 0$ then

$x-4=0$ or $x+5=0$

$\underline{\underline{x = 4}}$ $\qquad$ $\underline{\underline{x = -5}}$

Problem Determine the quadratic expression $x^2 + bx + c$ with the given roots (zeros)

a) 1 and 2

Solution: $(x-1)\cdot(x-2) = \underline{\underline{x^2 - 3x + 2}}$

b) 11 and -3

Solution: $(x-11)\cdot(x+3) = \underline{\underline{x^2 - 8x - 33}}$

Note $3(x-1)(x-2) = 3x^2 - 9x + 6$ has the same roots as $x^2 - 3x + 2$ (namely $x=1$ and $x=2$).

If r_1 and r_2 are the solutions ('roots') to the quad. eq.

$$x^2 + bx + c = 0$$

then $(x - r_1)\cdot(x - r_2) = x^2 - r_2 x - r_1 x + (-r_1)(-r_2)$

$$= x^2 - (r_1 + r_2)x + r_1 r_2$$

so $b = -(r_1 + r_2)$ and $c = r_1 r_2$

<u>EX</u> $x^2 + 6x - 16 = (x-2)(x+8)$ $\quad r_1 = 2$, $r_2 = -8$

<u>Problem</u> Solve the eq.

$$(x^2+1)(12+3x)(9-x^2)(x^2-3x+2) = 0$$

A product equal to zero: one of the factors has to be zero.

$x^2 + 1 = 0$ — no solutions

or $12 + 3x = 0$ $\quad 3(4+x) = 0$ so $\underline{x = -4}$

or $9 - x^2 = 0$ $\quad (3-x)(3+x) = 0$ so $\underline{x = \pm 3}$

or $x^2 - 3x + 2 = 0$ $\quad (x-1)(x-2) = 0$ so $\underline{x = 1}$ or $\underline{x = 2}$

or

<u>Problem</u> Solve the eq. $x^4 - 12\cdot x^3 + 11\cdot x^2 = 0$

<u>Solution</u> Factorise the LHS: $x^2(x^2 - 12x + 11) = 0$

$x^2 = 0$ or $x^2 - 12x + 11 = 0$

$\underline{\underline{x = 0}}$ $\quad (x-1)(x-11) = 0$

$\underline{\underline{x = 1}}$ or $\underline{\underline{x = 11}}$