EBA 1180, lecture 6, 12 Sept 2023, Runar Ile

Plan: 1. Repetition
2. Linear and quadratic equations
3. Equations with parametres: the abc-formula

1. Repetion

Ex (Course Paper 2021a, probl 1a)

i) calculate the sum

$$6000 \cdot 1.0025^{96} + 6000 \cdot 1.0025^{95} + \ldots + 6000 \cdot 1.0025^{26} + 6000 \cdot 1.0025^{25}$$

Solution This is a geometric series with

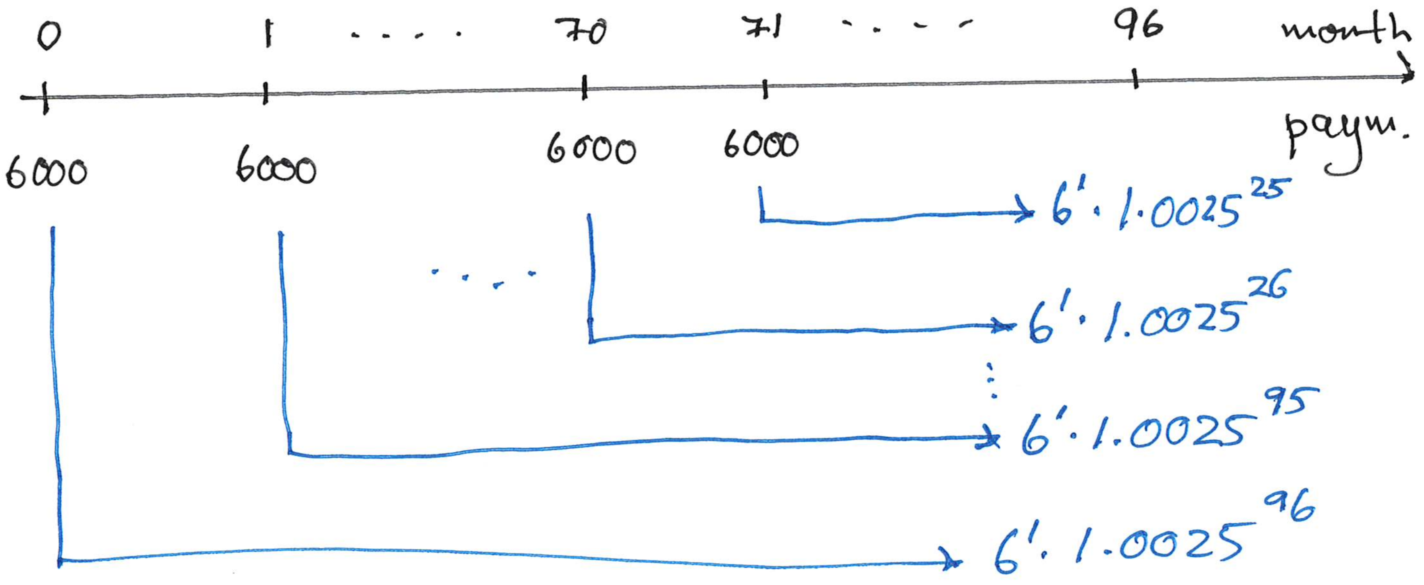
- the first term $= 6000 \cdot 1.0025^{25}$ (or: $6000 \cdot 1.0025^{96}$)
- the multiplicator ("growth factor") $= 1.0025$ (or: $\frac{1}{1.0025} = 1.0025^{-1}$)
- the number of terms $= 96 - 24 = 72$ (no choice!)

The formula for the sum is

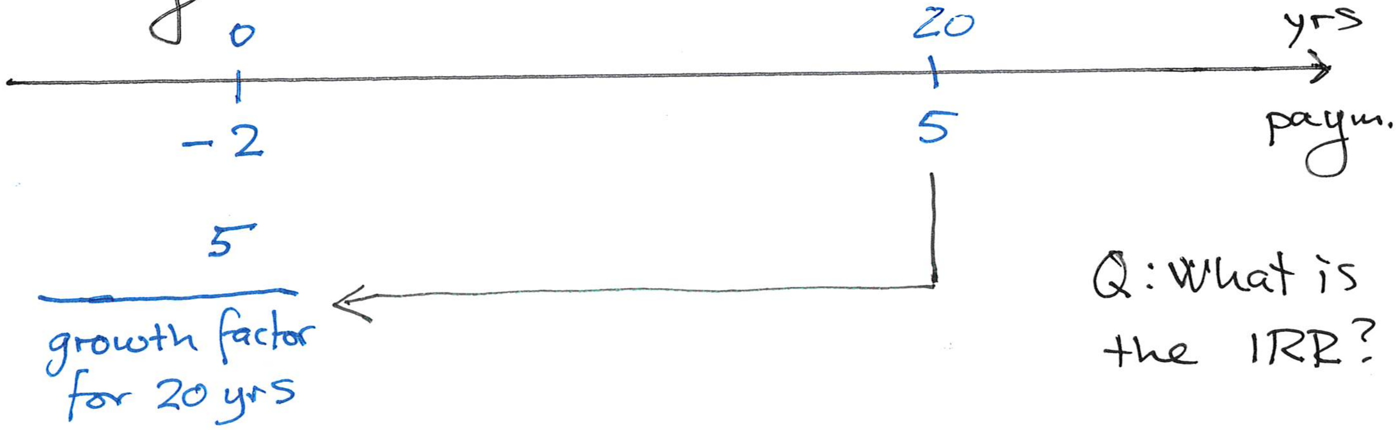
$$a_1 \cdot \frac{k^n - 1}{k - 1} = 6000 \cdot 1.0025^{25} \cdot \frac{1.0025^{72} - 1}{0.0025} = \underline{\underline{503122.08}}$$

ii) Decribe a financial situation where the sum is relevant (the important numbers should be interpreted).

Solution The sum read from left to right gives the account balance after 8 years when 6000 is deposited every month for 6 years (so 72 deposits), with the first deposit today (the first term in the sum), 3% nominal interest, monthly compounding with periode rate $\frac{3\%}{12} = 0.0025$



Probl 5 (last week) You consider investing 2 mill today to receive 5 mill. 20 yrs. from now.



Q: What is the IRR?

a) Annual compounding gives the eq (with r = IRR)

$$-2 + \frac{5}{(1+r)^{20}} = 0 \quad \text{so} \quad \frac{5}{(1+r)^{20}} = 2 \quad | \cdot (1+r)^{20}$$

$$5 = 2 \cdot (1+r)^{20} \quad : 2 \quad \text{and get} \quad (1+r)^{20} = \frac{5}{2}$$

$$\text{so} \quad 1+r = \sqrt[20]{\frac{5}{2}} = \left(\frac{5}{2}\right)^{\frac{1}{20}}$$

$$\text{and} \quad r = \left(\frac{5}{2}\right)^{\frac{1}{20}} - 1 = \underline{\underline{4.69\%}}$$

$2.5 \; \boxed{y^x} \; 20 \; \boxed{1/x} \; \boxed{-} \; 1 \; \boxed{=}$

b) With quaterly compounding and r = quaterly IRR gives the

eq. $\frac{5}{(1+r)^{80}} = 2$ (80 = 4·20 interest periods)

We get $r = \left(\frac{5}{2}\right)^{\frac{1}{80}} - 1 = 1.152\%$

Then the nominal internal rate of return is $4 \cdot 1.152\% = \underline{\underline{4.61\%}}$

d) With continuous compounding the annual growth factor is e^r (r = nominal IRR)

so the tot. present value of the cash flow is $-2 + \frac{5}{(e^r)^{20}}$ which

is supposed to be 0 which

gives the eq. $\frac{5}{(e^r)^{20}} = 2 \quad \Big| \cdot \frac{(e^r)^{20}}{2}$

so $(e^r)^{20} = \frac{5}{2}$ (or $e^{20r} = \frac{5}{2}$)

so $e^r = \left(\frac{5}{2}\right)^{\frac{1}{20}} = 1.0469$. We

~~try~~ different values of r.

A good answer is $r = \underline{\underline{4.58\%}}$

(or $r = \dfrac{\ln 5 - \ln 2}{20}$)

Start: 11.03

<u>Problem</u> You deposit 2 mill today, annual (<u>nominal</u>) interest is 12% with continuous compounding. Determine the balance after 1 yr and 7 months.

<u>Solution</u> $2 \text{ mill} \cdot \underbrace{e^{0.12}}_{1 \text{ yr}} \cdot \underbrace{\left(e^{0.12}\right)^{\frac{7}{12}}}_{7 \text{ months}}$ $\left(= \left(\underbrace{e^{\frac{0.12}{12}}}_{\text{growth factor for 1 month.}}\right)^{\overset{\#\text{months}}{19}} \right)$

$= 2 \text{ mill} \cdot e^{0.12} \cdot e^{0.12 \cdot \frac{7}{12}}$

$= 2 \text{ mill} \cdot e^{0.12\left(1 + \frac{7}{12}\right)} = 2 \text{ mill} \cdot e^{\cancel{0.12} \cdot \frac{19}{\cancel{12}}}$

$= 2 \text{ mill} \cdot e^{0.19} = \underline{\underline{2.4185 \text{ mill}}}$

$= \underline{\underline{2.42 \text{ mill}}}$

<u>2. Linear and quadratic equations.</u>

<u>A linear expression</u> $ax + b$ (a and b are numbers and $a \neq 0$)

<u>Ex</u> $4x - 3$ $(a = 4, \; b = -3)$

A linear equation An eq. which can be transformed into an equivalent equation of the form $ax + b = 0$ $(a \neq 0)$

EX The eq. $\frac{1}{x+3} = \frac{2}{x+4}$ $\mid \cdot (x+3)\cdot(x+4)$

multiply with a common denominator on each side.

is transformed to $x+4 = 2(x+3)$

We use the distributive law on the RHS.

$$x+4 = 2x+6$$

subtract $2x-6$ on each side

std. form: $-x-2 = 0$ $(a=-1, b=-2)$

$(x \neq -3,\ x \neq -4)$

A quadratic expression ax^2+bx+c

a, b, c are numbers and $a \neq 0$

A quad. eq. is an eq. that can be made into the eq. $ax^2+bx+c=0$

EX $\frac{1}{x} + \frac{2}{x+1} = 3$ $\mid \cdot x(x+1)$

$$x+1 + 2x = 3x(x+1)$$

$$3x+1 = 3x^2+3x$$

Subtract $3x+1$ on each side

$$3x^2-1 = 0 \quad (a=3,\ b=0,\ c=-1)$$

$(x \neq 0,\ x \neq -1)$

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3. Eq.s with paramtres: The abc-formula

If $a \neq 0$ then $ax^2+bx+c=0$ has solutions

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Ex $3x^2+4x-5=0$ $(a=3,\ b=4,\ c=-5)$

Then $x = \dfrac{-4 \pm \sqrt{4^2-4\cdot 3\cdot(-5)}}{2\cdot 3}$ $\qquad 76 = 4\cdot 19$

$$= \frac{-4\pm\sqrt{16+60}}{6} = \frac{-4\pm\sqrt{76}}{6}$$

$$= \frac{-4\pm\sqrt{4}\cdot\sqrt{19}}{6} = \frac{\cancel{2}(-2\pm\sqrt{19})}{\cancel{2}\cdot 3}$$

$$= \frac{-2\pm\sqrt{19}}{3} = -\frac{2}{3}\pm\frac{\sqrt{19}}{3}$$