Ex: $x + y = 4$
$x + ay = 6$

Let's solve via Cramer's rule!

$$A = \begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix}, \quad 2 \times 2 \text{ (square)}$$

$$\vec{b} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \quad |A| = a - 1$$

2 cases:

1) $a = 1$: $|A| = 0$

$\Rightarrow$ No solutions or infinitely many

2) $a \neq 1$: $|A| \neq 0 \Rightarrow$

One solution.

Use Cramer's rule to find it!

$$|A| = a - 1, \quad |A_1(\vec{b})| = \begin{vmatrix} 4 & 1 \\ 6 & a \end{vmatrix} = 4a - 6$$

$$|A_2(\vec{b})| = \begin{vmatrix} 1 & 4 \\ 1 & 6 \end{vmatrix} = 6 - 4 = 2$$

From Cramer's rule:

$$x = \frac{|A_1(\vec{b})|}{|A|} = \frac{4a-6}{a-1}$$

$$y = \frac{|A_2(\vec{b})|}{|A|} = \frac{2}{a-1}$$