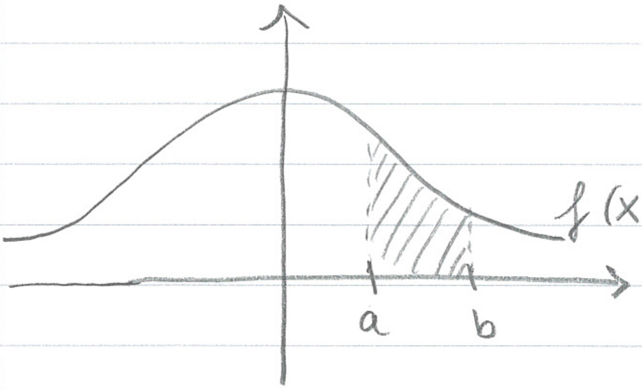
Other applications

Probabilities (continuous random variables):



$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, called the std. normal probability distribution. $X \sim N(0,1)$.

random variable

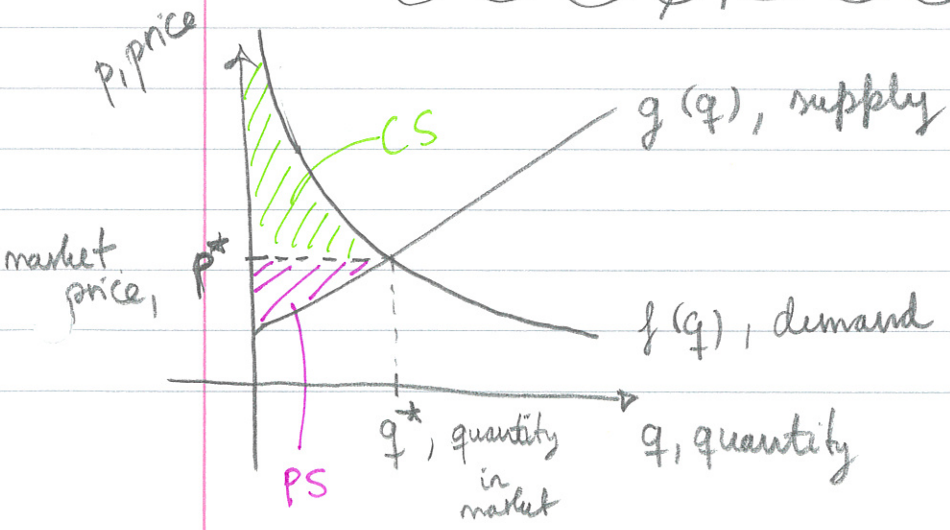
$\underbrace{P(a \leq X \leq b)}_{\text{Probability that the cont. r.v. } X \text{ takes a value in } [a,b].} = \int_a^b f(x)\,dx$

"instantaneous probability" $\rightsquigarrow$ corresponds to $P(X = x)$ for discrete random variables

EX: Height, temperature, stock price etc.

at a particular time

$\overbrace{\text{Consumer}}^{CS}$/$\overbrace{\text{producer surplus}}^{PS}$



• $p = f(q)$, demand function (inverse)

• $p = g(q)$, supply function (inverse)

CS : Consumer surplus

$$CS = \int_0^{q^*} f(q) - p^* \, dq$$

PS : Producer surplus

$$PS = \int_0^{q^*} p^* - g(q) \, dq$$

In the setting above:

Ex: $f(q) = \dfrac{100}{q+5}$, $g(q) = q + 5$

What is the market price?

$$f(q) = g(q)$$

Market price: When ~~[illegible]~~ demand = supply

$$\frac{100}{q+5} = q + 5$$

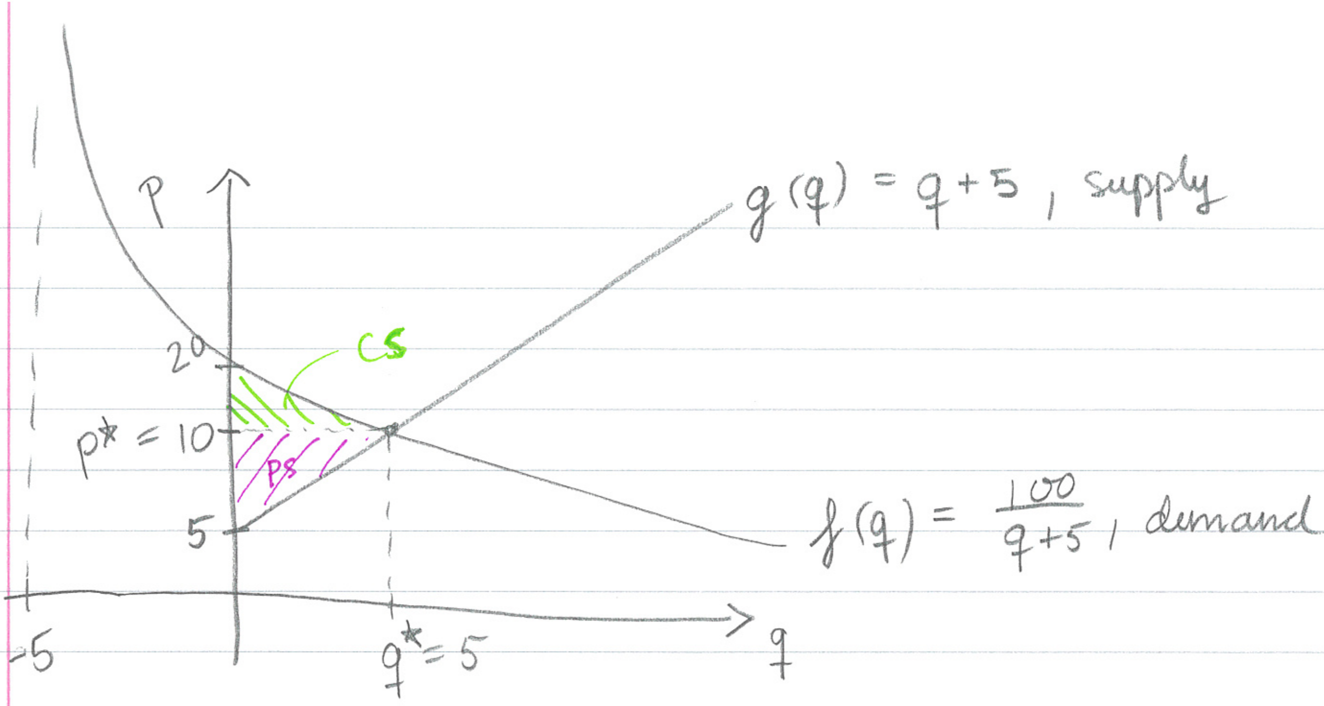
since ~~price~~ quant must be non-neg.

$$100 = (q+5)^2$$

$$q + 5 = \pm\sqrt{100} = \pm 10 = 10$$

$$q = 10 - 5 = 5$$

$$\underline{q^* = 5}$$



Producer surplus:

$$PS = \int_0^5 \underbrace{10}_{p^*} - g(q)\,dq = \int_0^5 10 - (q+5)\,dq$$

From formula

$$= \int_0^5 10 - q - 5\,dq$$

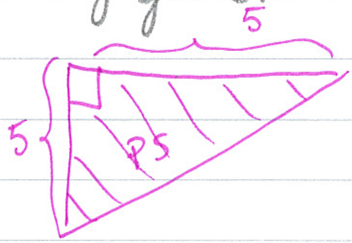
$$= \int_0^5 5 - q\,dq = \left[5q - \tfrac{1}{2}q^2\right]_{q=0}^{5}$$

$$= \left(5 \cdot 5 - \tfrac{1}{2}5^2\right) - \overbrace{\left(5 \cdot 0 + \tfrac{1}{2}0^2\right)}^{0}$$

$$= 25 - \frac{25}{2} = \underline{12{,}5}$$

Same!

NOTE: From figure:



Area of triangle $= PS = \frac{5 \cdot 5}{2} = \underline{12{,}5}$

$$CS = \int_0^5 \overbrace{\frac{100}{q-5}}^{f(q)} \overbrace{-10}^{p^*} \, dq$$

From formula

$$= \left[100 \ln(q+5) - 10q \right]_{q=0}^{5}$$

$$= 100 \ln 10 - 50 - (100 \ln(5) - 0)$$

$$= 100(\ln(10) - \ln(5)) - 50$$

$$= 100 \ln\left(\frac{10}{5}\right) - 50$$

$$= 100 \ln 2 - 50 \approx 19$$

$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

Exercises

$$\int 30 \underbrace{x \sqrt{x}}_{} \, dx = \int 30 \, x^{1+\frac{1}{2}} \, dx$$

TRICK: $x^1 \cdot x^{\frac{1}{2}}$

$$= \int 30 \, x^{\frac{3}{2}} \, dx$$

$$= 30 \int x^{\frac{3}{2}} \, dx$$

$$= 30 \, \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= 30 \, \frac{2}{5} \, x^{\frac{5}{2}} + C$$

$$= 12 x^{\frac{5}{2}} + C = 12 \, x^{2+\frac{1}{2}} + C$$

$$= 12 x^2 \sqrt{x} + C$$