EBA1180, lecture 3, 30 Aug. 2023, Runar Ile

Plan
1. A few examples
2. Total present value of a cash flow

1. A few examples

Problem The value of Kåre's flat inceases by 10% the first year and decreases by 30% the second year. Compute the relative change for the two years combined. (Hint: Answer is not −20%)

Solution

Relative change for the first year: $r_1 = 0.1$

——— " ——— second —"— : $r_2 = -0.3$

Rate of change for the first year: $1+r_1 = 1.1$

——— " ——— second —"— : $1+r_2 = 0.7$

——— " ——— for the two years combined:

$$(1+r_1)\cdot(1+r_2) = 1.1 \cdot 0.7 = 0.77$$

So the relative change for the two years is $0.77 - 1 = -0.23 = \underline{\underline{-23\%}}$

Pattern Relative changes of value: $r_1, r_2, \dots, r_n$ gives the combined relative change:

$$\underbrace{(1+r_1)\cdot(1+r_2)\cdot\ldots\cdot(1+r_n)}_{\text{combined rate of change}} - 1$$

Ex Deposit (principal): 50 000
Interest: $r = 4\%$ with annual compounding

After 5 years the balance is

$$50000 \cdot (1 + 4\%)^5 = \underline{\underline{60\,832.65}}$$

Calculator: 50 000 $\boxed{\times}$ 1.04 $\boxed{y^x}$ 5 $\boxed{=}$

Problem Deposit 50 000
Nominal interest 4 %
Monthly compounding.

a) Determine the balance after 5 years
b) Determine the effective interest

Solution a) After 5 years the balance is

$$50\,000 \cdot \left(1 + \frac{4\%}{12}\right)^{12 \cdot 5}$$

$$= 50\,000 \cdot \left(1 + \frac{0.04}{12}\right)^{60} = \underline{\underline{61\,049.83}}$$

b) Effective interest r_{eff} = the annual interest which gives the same balance as the period rate. (relative change for 1 year)

In (a) $1 + r_{eff} = \left(1 + \frac{0.04}{12}\right)^{12}$

$$= 1.040742$$

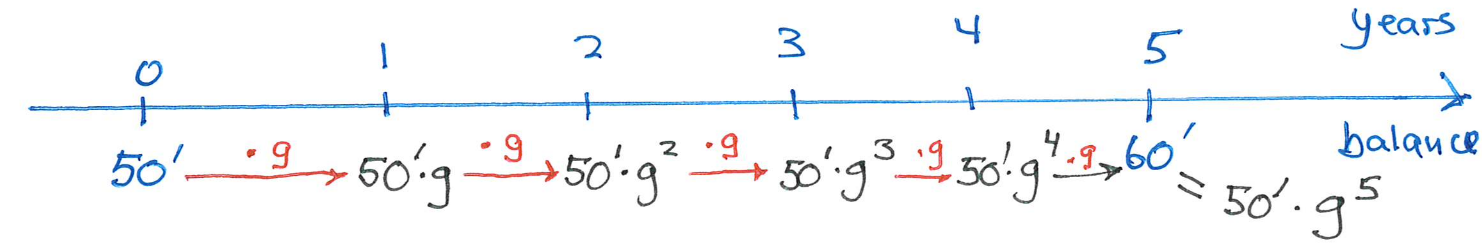
so $\underline{\underline{r_{eff} = 4.0742\%}}$

Start: 11.00

Problem After 5 years of added interest the deposit of 50 000 has become 60 000. Calculate the effective interest.

Solution The 5-year growth factor (rate of change) is

$$1 + \frac{60000 - 50000}{50000} = 1.2$$



Let g be the annual growth factor.

Then $50000 \cdot g^5 = 60000$

so $g^5 = \dfrac{60000}{50000} = 1.2$

so $g = (g^5)^{\frac{1}{5}} = 1.2^{\frac{1}{5}} \quad \left(= \sqrt[5]{1.2} \right)$

$= 1.2^{0.2} = 1.03714$

So $r_{eff} = 0.03714 = \underline{\underline{3.714\,\%}}$

2 Total present value of a cash flow

Present value of an amount (K) paid n years from now with interest r

= what you have to deposit today (K_0) for the balance to K n years from now with interest.

since $K = K_0(1+r)^n$ we get

$$K_0 = \frac{K}{(1+r)^n} \quad \text{(present value)}$$

EX 50000 (K) 3 years from now with 4 % interest has present value

$$K_0 = \frac{50\,000}{(1.04)^3} = \underline{\underline{44449.82}}$$

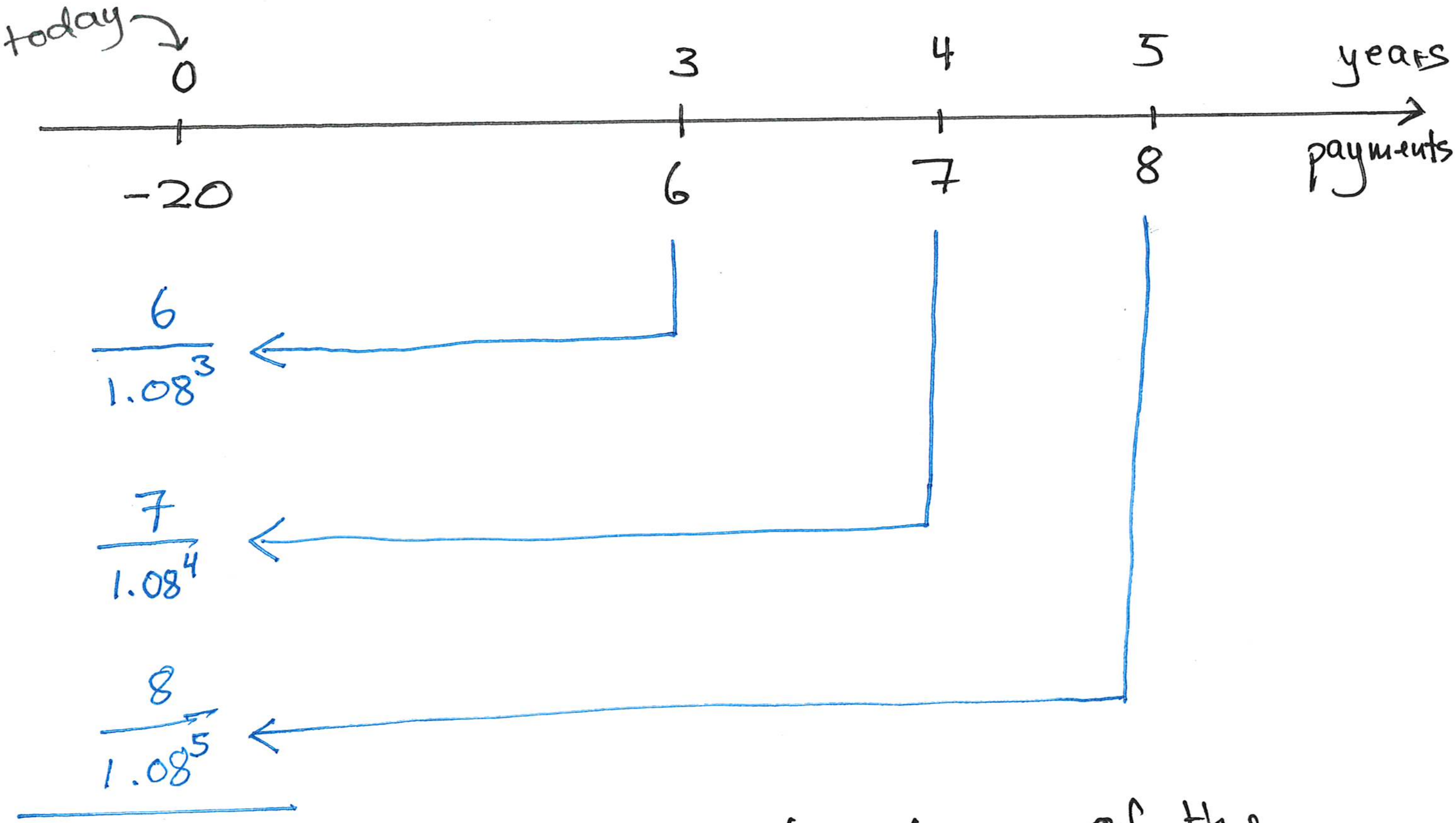
We can extend this to a cash flow (several payments combined)

EX You pay 20 mill. today, and get back

6 mill after 3 years
7 — " — 4 — " —
8 — " — 5 — " —

With 8% interest, what is the total present value of the cash flow?

It is the sum of the present values of each of the payments.



The sum = tot. present value of the cash flow

$$= -20 + \frac{6}{1.08^3} + \frac{7}{1.08^4} + \frac{8}{1.08^5} = \underline{\underline{-4.65}}$$

(investment not giving 8% annual yield)

The <u>internal rate of return</u> (IRR) is the interest which makes the total present value of cash flow equal to zero.

In general hard to calc. IRR. Here we have to solve the eq. $f(x) = -20 + \frac{6}{(1+x)^3} + \frac{7}{(1+x)^4} + \frac{8}{(1+x)^5}$
$= 0$

(x = interest)

Answer $x \approx 1.12\%$