Partial fractions

EBA 1180
Spring 24
Lecture 4
(28)

TRAN

Ex. continued:

Type iii): $\int \frac{2}{1-x^2}\,dx$

$$\frac{2}{1-x^2} = \frac{2}{(1-x)(1+x)} = \frac{A}{1+x} + \frac{B}{1-x}$$

$$0 \cdot x + 2 = (B-A)x + (A+B) \quad (\star)$$

Why OK to "compare coefficients" (like last lecture)?

Eq. $(\star)$ holds for all x, e.g. $x=0$:

$$\boxed{2 = A + B}$$

$$0 \cdot x + \cancel{2} = (B-A)x + \underbrace{\cancel{(A+B)}}^{2}$$

$$0 \cdot x = (B-A)x$$

Say $x=1$:

$$0 \cdot 1 = (B-A) \cdot 1$$

$$\boxed{0 = B - A}$$

$\Rightarrow A = B = 1$ partial fractions

$$\int \frac{2}{1-x^2}\,dx = \int \frac{1}{1+x} + \frac{1}{1-x}\,dx$$

$$= \int \frac{1}{1+x}\,dx + \int \frac{1}{1-x}\,dx$$

substitution

$$= \frac{1}{1}\ln|1+x| + \frac{1}{-1}\ln|1-x| + C$$

$$= \ln|1+x| - \ln|1-x| + C$$

$$= \ln\frac{|1+x|}{|1-x|} + C$$

Problem set 27

1) MET/EBA 1180 Spring 17

Ex. 1: $f(x) = 0{,}6 \ln(1+x) + 0{,}4 \ln(1-x)$,

$$0 \leq x < 1$$

a) $$f'(x) = 0{,}6 \frac{1}{1+x} \underbrace{\cdot 1}_{\text{from chain rule}} + 0{,}4 \frac{1}{1-x} \underbrace{(-1)}_{\text{from chain rule}}$$

$$= \frac{0{,}6}{1+x} - \frac{0{,}4}{1-x}$$

$$= \frac{0{,}6(1-x) - 0{,}4(1+x)}{(1+x)(1-x)}$$

$$= \frac{0{,}6 - 0{,}6x - 0{,}4 - 0{,}4x}{(1+x)(1-x)}$$

$$= \frac{0{,}2 - x}{(1+x)(1-x)}$$

So: $f'(x) = 0$ gives

$$\frac{0{,}2 - x}{(1+x)(1-x)} = 0$$

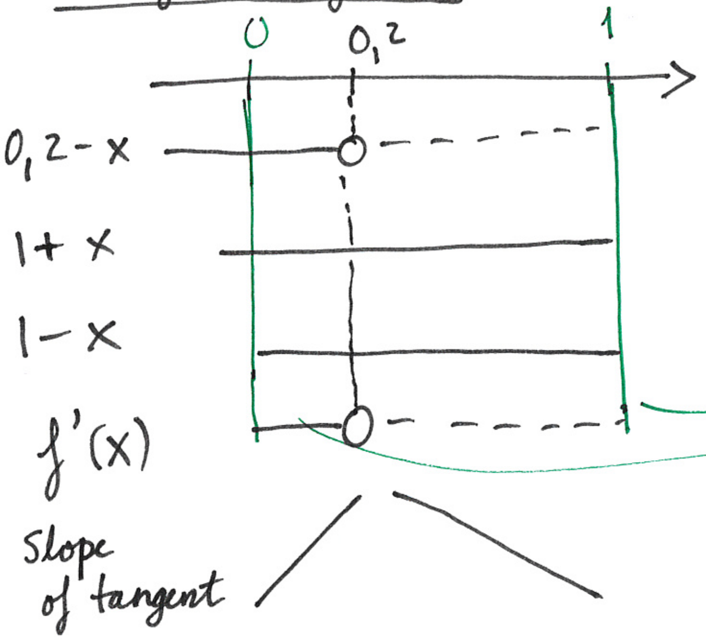
$$0{,}2 - x = 0$$

$$\underline{x = 0{,}2}$$

Candidate for the max. point: We need to check whether it actually is max. point.

To do so: Find sign of derivative

Sign diagram:



cap diagram between 0 and 1 (because of domain)

From this, our critical (candidate) point $x = 0{,}2$ is in fact a (global) max. point, so

$$\underline{x^* = 0{,}2}$$

The max. value:

$$f(x^*) = 0{,}6 \ln(1{,}2) + 0{,}4 \ln(0{,}8)$$

$$\approx \underline{0{,}0201}$$

calculator

b) $$f''(x) = \left(\underbrace{\frac{0,2-x}{1+x} \cdot \frac{1}{1-x}}_{f'(x)} \right)'$$

$$= \frac{(1+x)(-1)-(0,2-x)\cdot 1}{(1+x)^2}\;\frac{1}{1-x}$$

quotient rule combined with product rule

$$+ \frac{0,2-x}{1+x}\;\frac{1}{(1-x)^2}\underbrace{(-1)}_{\text{product rule + chain rule}}$$

$$= \frac{(-1-\cancel{x}-0,2+\cancel{x})(1-x)+(0,2-x)(1+x)}{(1+x)^2(1-x)^2}$$

$$= \frac{-1,2+1,2x+0,2+0,2x-x-x^2}{(1+x)^2(1-x)^2}$$

$$= \frac{-1+0,4x-x^2}{(1+x)^2(1-x)^2}$$

FOR ALL OLD EXAMS: dr-eriksen.no

Sign of numerator:

$$-x^2+0,4x-1=0 \quad \big|\cdot(-5)$$

$$5x^2-2x+5=0$$

Convex: $f''>0$

Concave: $f''<0$

abc-formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 5 \cdot 5}}{2 \cdot 5}$$

$$= \frac{2 \pm \sqrt{\underbrace{4 - 100}_{\text{Negative}}}}{10}$$

Negative

$\Downarrow$

$\boxed{\text{No real roots}}$

Alt: $\overbrace{x^2 - 0{,}4\,x + 1}$

So $-x^2 + 0{,}4x - 1$ is never 0.

Is it positive or negative?

$x = 0 \Rightarrow -0^2 + 0{,}4 \cdot 0 - 1 = -1 < 0$

$\hookrightarrow$ $\boxed{\text{Negative!}}$

Also $(1-x)^2 > 0$ and $(1+x)^2 > 0$, hence

$$f''(x) < 0 \quad \text{for all } x.$$

Hence, f is concave.

c) Show $f(x) < 0$ for $x > \overbrace{2 \cdot \overbrace{x^*}^{0,2}}^{0,4}$:

From a), $f'(x) < 0$ for $x > x^* = 0{,}2$ → See sign diagram

Hence, f is decreasing for $x > x^* = 0{,}2$.

Furthermore,

$$f(2x^*) = f(0{,}4) = 0{,}6 \ln(1{,}4) + 0{,}4 \ln(0{,}6)$$

$$\approx -0{,}0024 < 0$$

Hence, $f(\underbrace{2x^*}_{0,4}) < 0$ and $f(x)$ decreases for all $x > \underbrace{x^*}_{0,2}$, in particular for $x > \underbrace{2x^*}_{0,4}$

Therefore, $\underline{\underline{f(x) < 0}}$ when $x > \underbrace{2x^*}_{0,4}$.

d) <u>Sketch graph</u>: <u>We know</u>:

- $f(\underbrace{0,2}_{x^*}) \approx \underbrace{0,0201}_{\text{max value}}$ → From a)

- $f(\underbrace{0,4}_{2x^*}) \approx -0,0024$ → From c)

- f is increasing for $x < 0,2$ and decreasing for $x > 0,2$. → From a) (sign diagram)

- f is <u>concave</u> → From b)

- f is defined on $[0, 1)$ → From exercise
  $[0, 1\rangle$

<u>Where does f start?</u>

$$f(0) = 0,6 \ln \underbrace{(1+0)}_{0} + 0,4 \ln \underbrace{(1-0)}_{0} = 0$$

<u>What happens when we approach 1?</u>

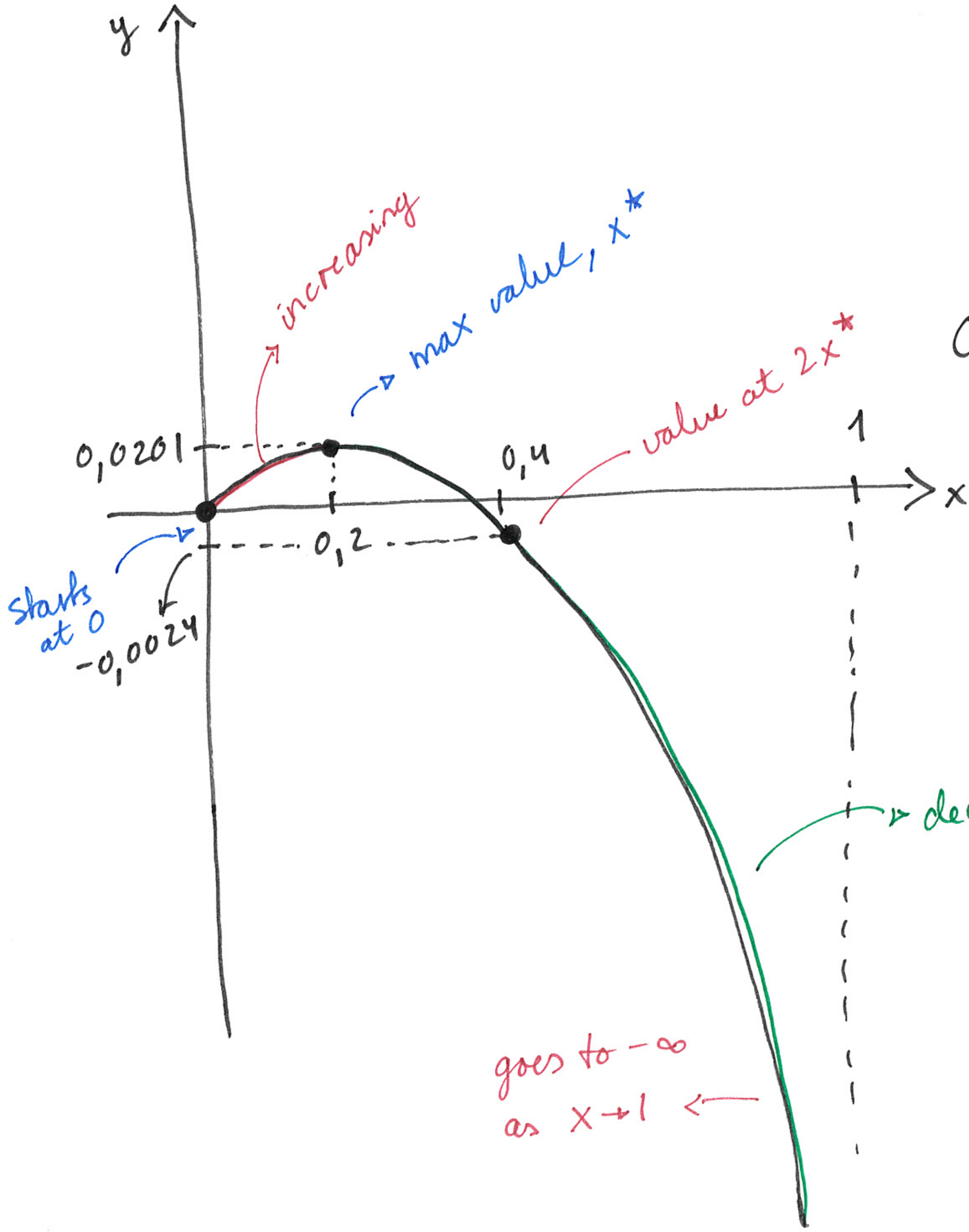
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \underbrace{0{,}6\ln(\underbrace{1+x}_{\to 2})}_{\to\, 0{,}6\ln(2)} + \underbrace{0{,}4\ln(\underbrace{1-x}_{\to 0})}_{\to\, -\infty}$$

$$= -\infty$$

"$0{,}6\ln 2 + (-\infty) = -\infty$"

DRAW ALL OF THIS !

$\boxed{D3-080}$



Concave (∩)