2) MET 11803 Fall 2018

$$f(x) = \frac{e^{1-\sqrt{x}}}{\sqrt{x}} \quad , x > 0$$

a) $f'(x) = ?$

$$f'(x) = \frac{\left(e^{1-\sqrt{x}}\right)' \cdot \sqrt{x} - e^{1-\sqrt{x}} \frac{1}{2\sqrt{x}}}{\left(\sqrt{x}\right)^2}$$

Quotient-rule

$$= \frac{\left(e^{1-\sqrt{x}}\right)' \sqrt{x} - \frac{e^{1-\sqrt{x}}}{2\sqrt{x}}}{x}$$

$$= \frac{\overbrace{e^{1-\sqrt{x}}}^{g'(u)} \overbrace{\left(-\frac{1}{2\cancel{\sqrt{x}}}\right)}^{u'} \cancel{\sqrt{x}} - \frac{1}{2\sqrt{x}} e^{1-\sqrt{x}}}{x}$$

Chain rule:

$u = 1 - \sqrt{x}$

$u' = -\frac{1}{2\sqrt{x}}$

Recall: $\sqrt{x} = x^{\frac{1}{2}}$, then regular rule for diff'ing powers

$$= \frac{e^{1-\sqrt{x}}\left(-\sqrt{x} - 1\right)}{2x\sqrt{x}}$$

$$\underbrace{e^{u}}_{g(u)} \Rightarrow g'(u) = e^{u}$$

b) Show that f is decreasing in $D_f = (0,\infty)$:

Since f decreasing $\Leftrightarrow f' < 0$

Suffices to show that $f'(x) < 0$ for $x \in (0,\infty)$.

element in

Consider the expression in a) for f': When $x > 0$, we see that

Denominator: $\underbrace{\underbrace{x}_{+}\underbrace{\sqrt{x}}_{+}}_{+} > 0 \Rightarrow$ Positive denominator

Numerator: • $e^{\overset{+}{u}} > 0$ for all u, in particular $\underbrace{e^{1-\sqrt{x}}}_{+} > 0$ for all $x > 0$.

• $\underbrace{-\underbrace{\sqrt{x}}_{+}}_{\text{neg.}} - 1 < 0$ for $x > 0 \Rightarrow$ Negative numerator

neg. $- 1 < 0$

Hence, $f'(x) < 0$ for all $x \in (0,\infty)$, so f is decreasing in D_f.

$f' = \frac{\text{neg.}}{\text{pos.}} = \text{neg.}$

c) Determine the limits

$$\lim_{x \to 0^+} f(x) \quad \text{and} \quad \lim_{x \to \infty} f(x):$$

x goes to 0 from above:

$0 \quad 1 \quad 2 \quad x$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{e^{1-\sqrt{x}}}{\sqrt{x}}$$

$e^{1-\sqrt{x}} \to e^{1-\sqrt{0}} = e^1 = e$

$\sqrt{x} \to \sqrt{0^+} = 0^+$

$\to$ "$\frac{e}{0^+}$", ∞ $+\infty$

$$= \underline{\underline{\infty}}$$

Ex:

$\frac{2{,}71}{0{,}1} = 27{,}1$

$\frac{2{,}71}{0{,}01} = 271$

etc.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{e^{1-\sqrt{x}}}{\sqrt{x}}$$

$e^{1-\sqrt{x}} \to$ "$e^{1-\sqrt{\infty}} = e^{-\infty} = \frac{1}{e^{\infty}} = 0$"

$\sqrt{x} \to$ "$\sqrt{\infty} = \infty$"

$\to$ "$\frac{0}{\infty}$" $= 0$

$$= \underline{\underline{0}}$$

d) Sketch the graph based on what we have found out and mark the area between the graph of f and the x-axis (for $x > 0$) in the sketch:

b)

We know: • f is decreasing on $(0, \infty) = D_f$

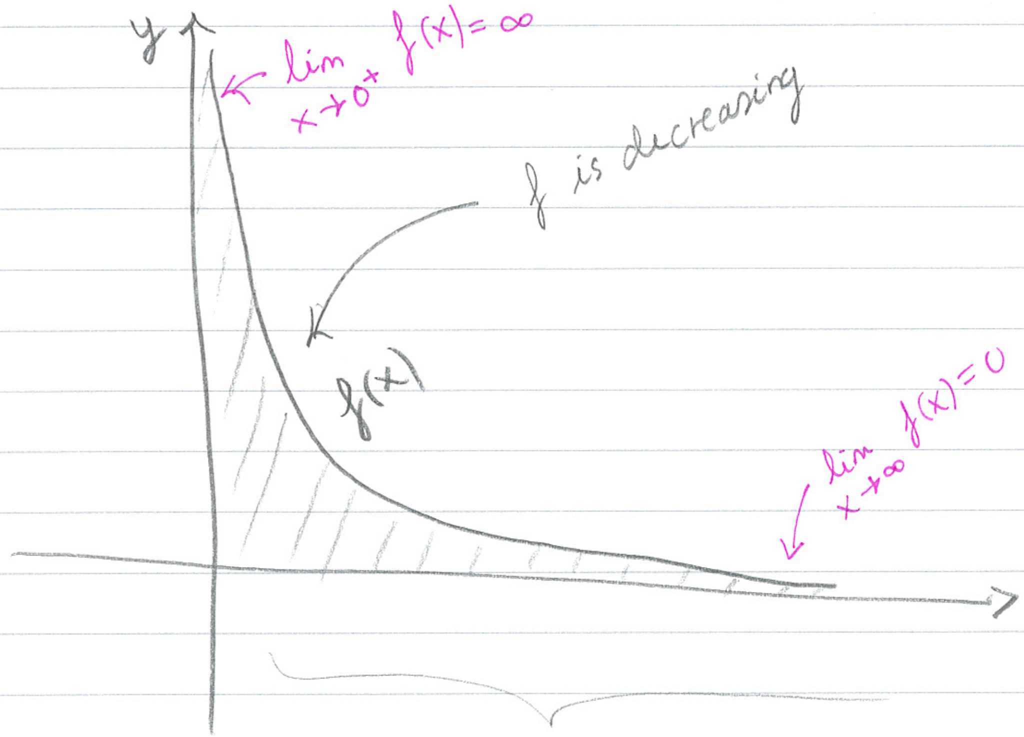
So f is only defined for $x \in (0, \infty)$

• $\lim\limits_{x \to 0^+} f(x) = \infty$

$\lim\limits_{x \to \infty} f(x) = 0$

From c)

Let's draw these facts!



f is defined for $x \in (0, \infty)$.

The area in question is marked.