

- Plan
1. Repetition: Elasticity
 2. Linear approximation
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1. Rep: Elasticity $p = \text{price/unit}$, demand function $D(p)$.

Ex $D(p) = 200 \cdot e^{-0.01p}$

Calculate $E(p)$ - the elasticity function.

We have $D'(p) \stackrel{\text{chain rule}}{=} -0.01 \cdot 200 \cdot e^{-0.01p} = -2e^{-0.01p}$

$$\text{So } E(p) = \frac{D'(p) \cdot p}{D(p)} = \frac{-2e^{-0.01p} \cdot p}{200 \cdot e^{-0.01p}} = \underline{\underline{-0.01p}}$$

The demand is elastic w.r.t. price if

$$E(p) < -1, \text{ that is } -0.01p < -1 \text{ (an inequality!)}$$

that is: $p > 100$

Meaning if $p > 100$, then a small increase in the price gives a decline in revenue.

Ex $E(110) = -1.1$, so a price increase of 1% from 110 gives a demand decline of 1.1%

The demand is inelastic w.r.t. price if

$$E(p) > -1, \text{ that is } -0.01p > -1, \text{ so}$$

$p < 100$

Meaning If $p < 100$, a small price increase gives an increase in revenue.

Ex $\epsilon(80) = -0.8$, so 1% price increase from $p = 80$ gives 0.8% demand decrease

If $\epsilon(p) = -1$ (so $p = 100$) then demand is unit elastic w.r.t. price.

Meaning: No (or very little) change in revenue if price is changed a little from $p = 100$.

2. Linear approximation

Ex $f(x) = \sqrt{x}$

The linear approximation of $f(x)$ about $x = 1$

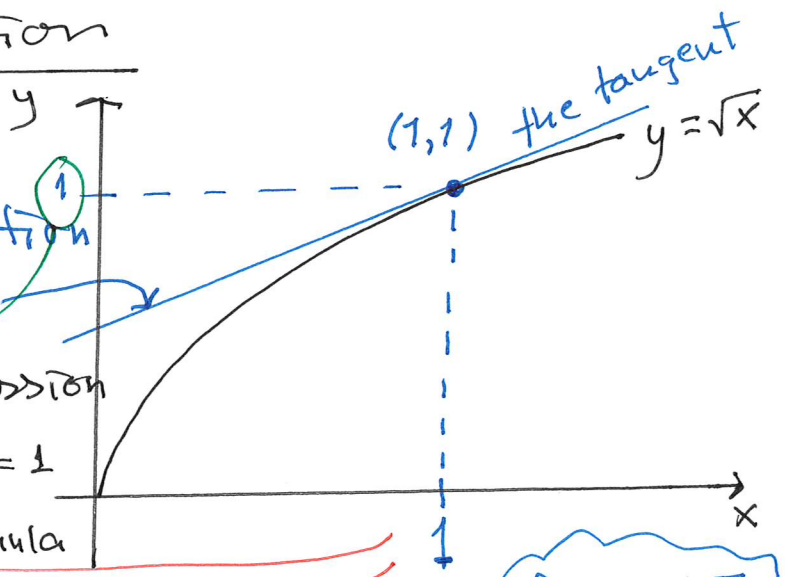
We can find the expression for the tangent line at $x = 1$ by the point-slope formula

$$y - 1 = f'(1) \cdot (x - 1)$$

so $y - 1 = \frac{1}{2}(x - 1)$

or $y = 1 + \frac{1}{2}(x - 1) = P_1(x)$ notation

- is called the degree one Taylor polynomial of \sqrt{x} about $x = 1$



$$\begin{aligned} f(x) &= \sqrt{x} \\ &= x^{1/2} \\ f'(x) &= \frac{1}{2} \cdot x^{-1/2} \\ &= \frac{1}{2\sqrt{x}} \\ f'(1) &= \frac{1}{2\sqrt{1}} = \frac{1}{2} \end{aligned}$$

$$\underline{\text{Ex}} \quad P_1(1.1) = 1 + \frac{1}{2}(1.1-1) = 1.05$$

(check: $\sqrt{1.1} = 1.04881\dots$)

3. Higher degree Taylor polynomials

$$\underline{\text{Ex}} \quad f(x) = \sqrt{x}$$

The Taylor polynomial of degree 2 for \sqrt{x}

about $x=1$ is

$$P_2(x) = \underbrace{f(1) + f'(1) \cdot (x-1)}_{P_1(x)} + \frac{f''(1)}{2} (x-1)^2$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$$

Pattern

$$P_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

- the second degree Taylor polynomial of a function $f(x)$ about $x=a$.

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f''(x) = \frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot x^{-\frac{1}{2}-1}$$

$$= -\frac{1}{4} \cdot x^{-\frac{3}{2}}$$

$$= -\frac{1}{4x\sqrt{x}}$$

$$f''(1) = -\frac{1}{4 \cdot 1 \cdot \sqrt{1}} = -\frac{1}{4}$$

$$\sqrt{2} = f(2) \approx P_2(2) = 1 + \frac{1}{2}(2-1) - \frac{1}{8}(2-1)^2$$

$$= 1 + \frac{1}{2} - \frac{1}{8} = 1.375$$

(check: $\sqrt{2} = 1.41421\dots$)

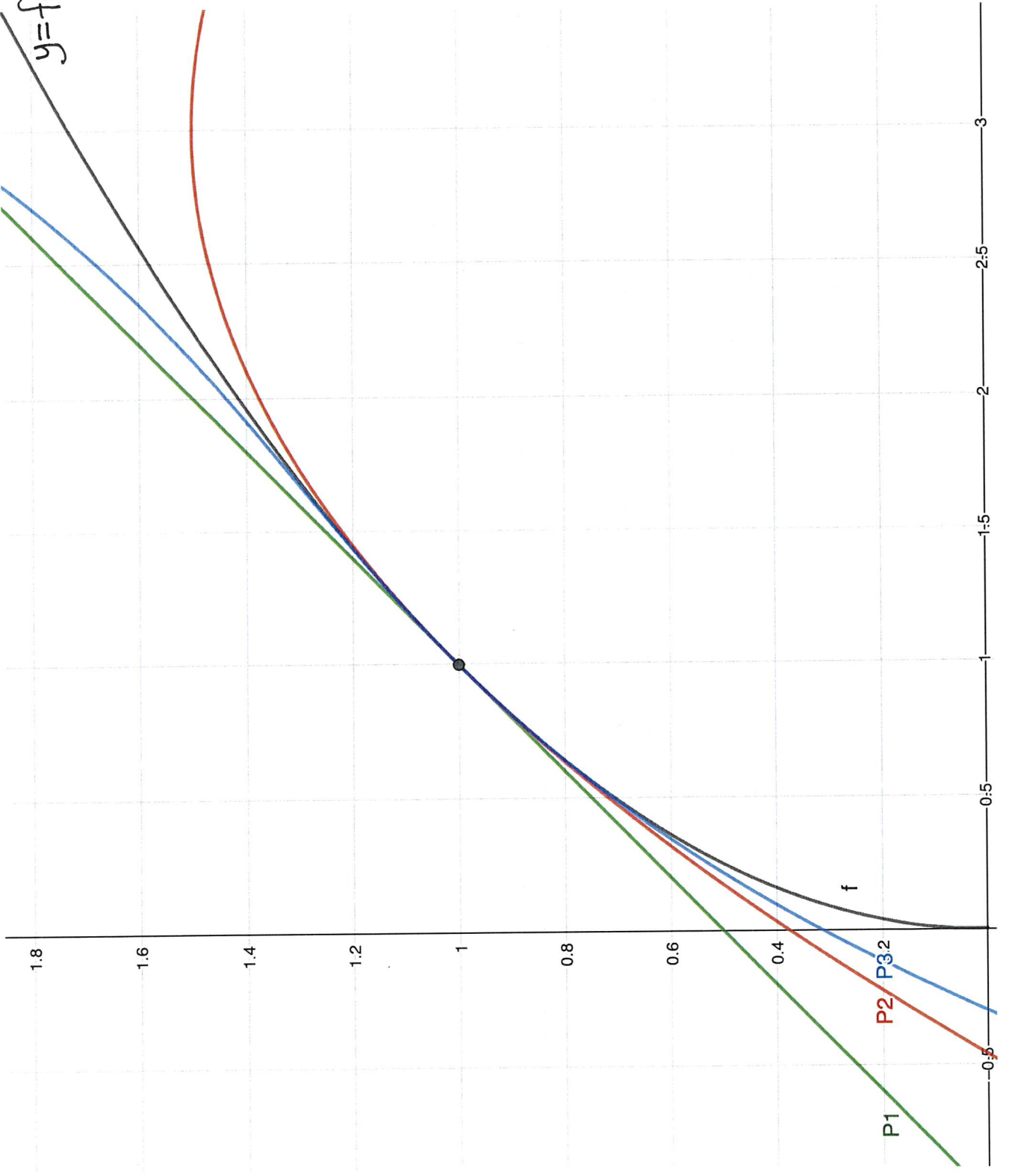
$$P_2(1.2) = 1 + \frac{1}{2}(1.2-1) - \frac{1}{8}(1.2-1)^2$$

$$= 1 + 0.1 - 0.005 = 1.0950$$

(check: $\sqrt{1.2} = 1.0954\dots$)

start: 11.02

$$y = f(x) = \sqrt{x}$$



Ex $f(x) = \sqrt{x}$ about $x=1$

Then the third degree Taylor polynomial for $f(x)$ about $x=1$ is:

$$P_3(x) = P_2(x) + \frac{f'''(1)}{6} \cdot (x-1)^3$$

already done this!

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{\left(\frac{3}{8}\right)}{6}(x-1)^3$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$$

$$P_3(1.2) = 1 + \frac{1}{2}(1.2-1) - \frac{1}{8}(1.2-1)^2 + \frac{1}{16}(1.2-1)^3$$

$$= 1.0955$$

$$\begin{aligned} f''(x) &= -\frac{1}{4} \cdot x^{-\frac{3}{2}} \\ f'''(x) &= -\frac{1}{4} \cdot \left(-\frac{3}{2}\right) \cdot x^{-\frac{5}{2}} \\ &= \frac{3}{8x^2\sqrt{x}} \\ f'''(1) &= \frac{3}{8 \cdot 1^2 \cdot \sqrt{1}} \\ &= \frac{3}{8} \end{aligned}$$

Pattern (3rd degree Taylor polynomial about $x=a$)

$$P_3(x) = \underbrace{f(a) + f'(a)(x-a)}_{P_2(x)} + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{6}(x-a)^3$$

The degree n Taylor polynomial for $f(x)$ about $x=a$:

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

where $n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$

4. About the exam

- 12 problems of equal weight
 - might have subproblems
- 3 hours (14-17), find out where!
- You write your answers on paper!
(3 layers! - cross over, not erase!)
Advice: One problem per page.
- I am grading.
- The exam counts 40% of the final grade.
- All the problems should be (very) recognizable from tutoring and the lectures.
- Many basic and central problems.
- The problems are not ordered according to the lecture plan.
- Support materials: BI-calculator, Ruler.

5. How to prepare

① Relevant material:

- lec. notes
- tutoring problems
- earlier (multiple choice) exams.
- also textbook problems

② My best advice:

Try to "solve" the problems in your head!

- what is the plan (in detail)
- what kind of knowledge is required?
- what kind of obstacles may occur?

③ If I get the wrong answer:

- what went wrong? - the plan
- the calculations

④ When you have solved a problem

- what did you learn

⑤ Learn the basics well!

- definitions, concepts ("the words")

⑥ Basic problems are the most important.

Ex $e^x = 5$ or $\ln(x+3) = 2$