EBA 1180, lecture 23, 14 Nov. 2023, Runar Ile

Plan: 1. Repetition: l'Hôpital's rule (probl. 1h)
Cost functions (probl. 3c)

2. Elasticity

---

1. Repetition l'Hopital's rule: For limits $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

- Differentiate the numerator and the denominator seperately
- Consider the same limit of the new fraction

Probl. 1h

$$\lim_{x\to 1} \frac{\ln(x)}{\sqrt{x}-1} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{\text{l'Hôp.}}{=}} \lim_{x\to 1} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{2\sqrt{x}}\right)} = \frac{\left(\frac{1}{1}\right)}{\left(\frac{1}{2\sqrt{1}}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = \underline{\underline{2}}$$

Meaning: $\frac{\ln(x)}{\sqrt{x}-1}$ has <u>no</u> vertical asymptote for $x=1$

Also: (extra!)

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}-1} \underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{\text{l'Hôp}}{=}} \lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{2\sqrt{x}}\right)} \quad \Big| \cdot \frac{x\cdot 2\sqrt{x}}{x\cdot 2\sqrt{x}} = 1$$

$$= \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to\infty} \frac{2\cancel{\sqrt{x}}}{\sqrt{x}\cdot\cancel{\sqrt{x}}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = \underline{\underline{0}}$$

Meaning: $\frac{\ln(x)}{\sqrt{x}-1}$ has horizontal asymptote $y=0$

cost functions

Probl. 3c $C(x) = 400 \cdot e^{0.001x^2}$ $(x \geq 0)$ is a cost function because

(1) $C(0) = 400 \cdot e^{0.001\cdot 0^2} = 400 \cdot e^0 = 400 > 0$

(2) $C'(x) \overset{\text{chain rule}}{=} 400 \cdot 0.001 \cdot 2x \cdot e^{0.001\cdot x^2}$

$= 0.8 \cdot x \cdot e^{0.001\cdot x^2} \geq 0$ for $x \geq 0$

(3) $C''(x) \overset{\text{prod. rule + chain rule}}{=} 0.8 \cdot e^{0.001 \cdot x^2} + 0.8x \cdot 0.002 \cdot x\, e^{0.001x^2}$

$$= \underbrace{0.8}_{>0} \cdot \underbrace{(1 + 0.002x^2)}_{>0} \underbrace{e^{0.001 \cdot x^2}}_{>0} > 0 \quad \text{for all } x$$

So $C(x)$ is a cost function which is strictly convex.

Then the cost optimum is the solution of the eq.

$$C'(x) \overset{\text{nice result}}{=} A(x) \qquad \left(A(x) = \frac{C(x)}{x}\right)$$

that is $0.8xe^{0.001x^2} = \frac{400\, e^{0.001x^2}}{x}$ $\quad | \cdot x$

gives $0.8 \cdot x^2 e^{0.001x^2} = 400 \cdot e^{0.001x^2}$ $\quad | : e^{0.001x^2}$

$$0.8x^2 = 400$$

$$x^2 = \frac{400}{0.8} = 500$$

So $x = \underline{\underline{\sqrt{500}}} = \underline{\underline{22.36}}$ $\quad (x \geq 0)$

is the cost optimum.

The minimal average unit cost is

$$A(\sqrt{500}) \overset{\text{by nice result}}{=} C'(\sqrt{500}) = 0.8 \cdot \sqrt{500} \cdot e^{0.001 \cdot (\sqrt{500})^2}$$

$$= \underline{\underline{29.49}}$$

2. Elasticity

p = price/unit

$D(p)$ = demand of a commodity with price p
= # units sold

The problem of comparing units.

Ex A barrel of North Sea crude oil costs $82.53

A litre of —"— costs NOK 5.32

The price elasticity of the demand is

$$\varepsilon = \frac{\text{relative change in demand}}{\text{relative change in price}}$$

← these numbers are independent of units

Ex In a month the price of a commodity drops from 12 thousand to 10 thousand and the demand increases from 50 mill. to 60 mill. Then

$$\varepsilon = \frac{\left(\frac{60-50}{50}\right)}{\left(\frac{10-12}{12}\right)} = \frac{\left(\frac{10}{50}\right)}{\left(\frac{-2}{12}\right)} = \frac{120}{-100} = -1.2$$

Interpretation If the price increases by 1% then the demand decreases by 1.2%.

Start: 11.02

Theory Assume we have a demand function $D(p)$. If the price is changed from p to $p+h$, the relative change in price is $\frac{p+h-p}{p} = \frac{h}{p}$. Then

$$\frac{\text{relative change in demand}}{\text{relative change in price}}$$

$$= \frac{\left(\frac{D(p+h)-D(p)}{D(p)}\right)}{\left(\frac{h}{p}\right)} \qquad \Big| \cdot \frac{p \cdot D(p)}{p \cdot D(p)} = 1$$

$$= \frac{D(p+h)-D(p)}{h} \cdot \frac{p}{D(p)}$$

$\downarrow h \to 0$ (the price change approaches 0)

$$\mathcal{E}(p) = D'(p) \cdot \frac{p}{D(p)}$$

This is the momentary price elastisity of the demand function.

Ex $D(p) = 50-p$ for $0<p<50$

Then $D'(p) = -1$ and $\mathcal{E}(p) = \frac{(-1)\cdot p}{50-p}$

$$= \frac{-p}{50-p}$$

Important question

Is the revenue going up or down if we increase the price a little?

Revenue $R(p) = p \cdot D(p)$

- is $R(p)$ increasing or decreasing?

$$R'(p) \overset{\text{prod. rule}}{=} 1 \cdot D(p) + p \cdot D'(p)$$

$$= D(p) \cdot \left[1 + \frac{p \cdot D'(p)}{D(p)} \right]$$

$$= \underbrace{D(p)}_{\text{always pos.}} \cdot \underbrace{\left[1 + \varepsilon(p) \right]}_{\text{pos. or neg.?}}$$

If $\varepsilon(p) < -1$
we get neg. $R'(p)$
so $R(p)$ is decreasing

Say: elastic demand

If $\varepsilon(p) > -1$
we get pos. $R'(p)$
so $R(p)$ is Increasing

Say: inelastic demand

If $\varepsilon(p) = -1$
then $R'(p) = 0$ and $R(p)$ has a stationary point.

say: the demand is unit elastic

Ex $D(p) = 50 - p \quad (0 < p < 50)$. We got

$$\varepsilon(p) = \frac{-p}{50-p}.$$

Q: In what price range do we have elastic demand?

Solve the inequality: $\varepsilon(p) < -1$

so $\frac{-p}{50-p} < -1 \quad | +1$

get $\frac{-p}{50-p} + 1 < 0$

so $\frac{-p + 50 - p}{50 - p} < 0$

so $\frac{50 - 2p}{\underbrace{(50 - p)}_{\text{pos. since } p < 50}} < 0$

so $50 - 2p < 0$ so $50 < 2p$

and $\underline{p > 25}$

Elastic demand w.r.t. price for $p \in \langle 25, 50 \rangle$

Inelastic ——— " ——— $p \in \langle 0, 25 \rangle$

Unit elastic ——— " ——— $p = 25$