EBA 1180, Lecture 22, 8 Nov. 2023, Runar Ile

Plan: 1. Marginal cost, revenue, profit...
2. Average unit cost and costoptimum

1. Marginal cost, revenue, profit ...

Intro: Diamonds and water

Ex: Cost of removing x % of pollution from a lake

$C(x)$ is the total cost of producing x units (of some commodity)

$C'(x)$ is (by definition) the marginal cost at x.

Interpretation The cost of producing one more unit than x units.

$$= C(x+1) - C(x) = \frac{C(x+1)-C(x)}{1} \approx \lim_{h\to 0} \frac{C(x+h)-C(x)}{h} = C'(x)$$

Why $C'(x)$? – much simpler math to work with!

$R(x)$ is the total revenue of selling x units.

$R'(x)$ is the marginal revenue of x.

Ex x = tons of salmon produced and sold

$R'(50) \approx$ extra revenue from selling 51 tons instead of 50 tons.
$= R(51) - R(50)$

The profit function: $P(x) = R(x) - C(x)$

$\Pi(x)$ – the economists

$P'(x) = R'(x) - C'(x)$
is the marginal profit function

2. Average unit cost and cost optimum

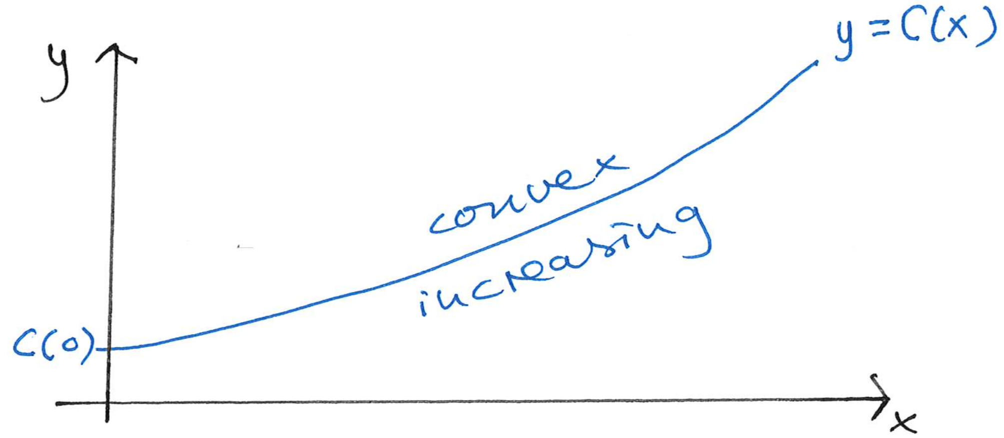
Average unit cost producing x units is $A(x) = \dfrac{C(x)}{x}$ – not a constant function!

<u>Definition</u> $C(x)$ is a cost function if

① $C(0) > 0$ (start-up cost)

② $C(x)$ is increasing ($C'(x) \geq 0$)

③ $C(x)$ is convex ($C''(x) \geq 0$)



<u>Definition</u> If $x = c$ is the minimum point for $A(x)$, then c is called the cost optimum (the x-value that gives the minimal average unit cost)

$$\boxed{\begin{array}{l}\text{Result: If } C(x) \text{ is a cost function with } C''(x) > 0 \text{ for } x > 0, \text{ then the cost optimum is the solution of the equation} \\ \qquad C'(x) = A(x)\end{array}}$$

Ex $C(x) = x^2 + 200x + 160\,000$.

This is a cost function because:

① $C(0) = 160\,000 > 0$

② $C'(x) = 2x + 200 > 0$ for $x \geq 0$

③ $C''(x) = 2 > 0$ for all x

By the result the cost optimum is the solution of the equation

$$C'(x) = A(x)$$

$$2x + 200 = \frac{x^2 + 200x + 160\,000}{x}$$

$$\cancel{2}x + \cancel{200} = \cancel{x} + \cancel{200} + \frac{160\,000}{x}$$

$$x = \frac{160\,000}{x} \quad \Big| \cdot x$$

$$x^2 = 160\,000$$

So $\underline{\underline{x = 400}}$ (only pos. x).

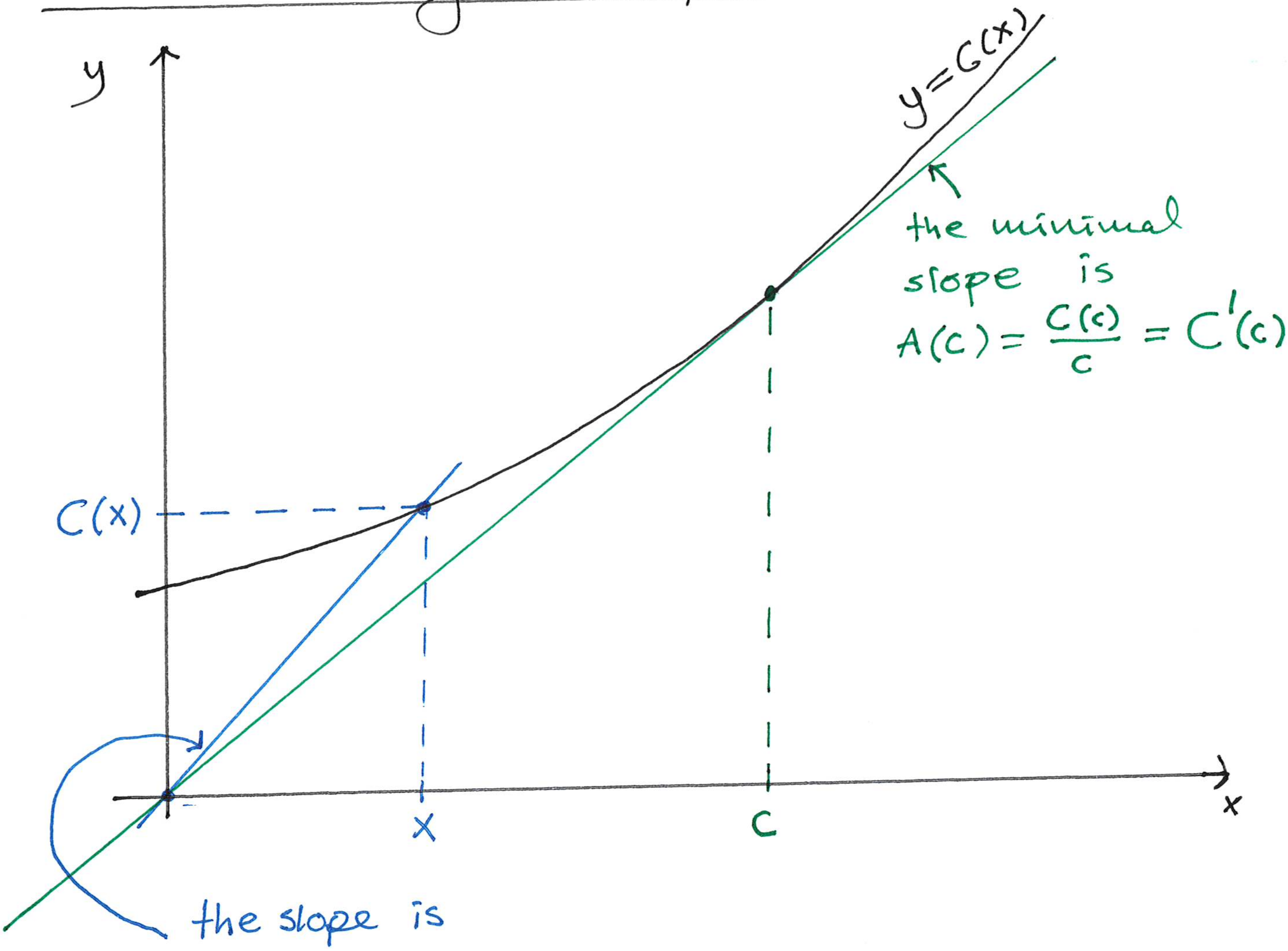
is the cost optimum (by the result).

The minimal average unit cost is then

$$A(400) = C'(400) = 2 \cdot 400 + 200 = \underline{\underline{1000}}$$

Start: 11.01

Geometric argument for the result



$\frac{C(x)}{x} = A(x)$ and $A(c) = \frac{C(c)}{c}$ is

the minimal unit cost when $C'(c) = A(c)$

= the smallest slope the origin

= the slope of the tangent which goes through the origin.

Algebraic reason for the result

We determine the stationary point of $A(x)$. Calculate

$$A'(x) = \left[\frac{C(x)}{x}\right]' \underset{\text{fract. rule}}{=} \frac{C'(x)\cdot x - C(x)\cdot 1}{x^2} \qquad \Big| \; \frac{:x}{:x}$$

$$= \frac{C'(x) - A(x)}{x}$$

So $A'(x) = 0$ is equivalent to $C'(x) \overset{(*)}{=} A(x)$.

Assume $x = c$ is such a stationary point, i.e. a solution of $(*)$. We use the second derivative test:

If $A''(c) > 0$ then c is a (loc) min. point.

Calculate:

$$A''(x) = \frac{[C''(x) - A'(x)]\cdot x - [C'(x) - A(x)]\cdot 1}{x^2}$$

Substitute $x = c$:

$$A''(c) = \frac{[C''(c) - \underbrace{A'(c)}_{=0}]\cdot \cancel{c} - [\underbrace{C'(c) - A(c)}_{=0}]}{c^{\cancel{2}}}$$

$$= \frac{C''(c)}{c} > 0 \quad (\text{for } c > 0).$$