EBA 1180, lecture 2, 29 Aug. 2023, Runar Ile

Plan
1. Repetition (alg. exp., roots & powers, absolute value)
2. Relative change and rate of change
3. Interest
4. Present value

---

1. Repetition

Fractions $\frac{2}{3} \cdot \frac{5}{4} = \frac{2 \cdot 5}{3 \cdot 4} = \frac{10}{12}$

and $\frac{x+3}{x+4} \cdot \frac{x-1}{x+2} = \frac{(x+3)\cdot(x-1)}{(x+4)\cdot(x+2)}$

Probl. 1 i $\frac{18}{4} \cdot \frac{\frac{2}{3}}{12} = \frac{18 \cdot \frac{2}{3}}{4 \cdot 12} = \frac{\frac{18}{1} \cdot \frac{2}{3}}{4 \cdot 12}$

$= \frac{\frac{18 \cdot 2}{1 \cdot 3}}{4 \cdot 12} = \frac{\frac{18 \cdot 2}{3} \cdot 3}{4 \cdot 12 \cdot 3} = \frac{18 \cdot 2}{4 \cdot 12 \cdot 3} = \frac{9 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$

$= \underline{\underline{\frac{1}{4}}}$

Probl 2 i $\frac{x^2 - 3x}{x(y-3)} \cdot \frac{xy^2 - 9x}{x-3} = \frac{x(x-3)}{x(y-3)} \cdot \frac{x(y^2-9)}{x-3}$

$= \frac{(x-3)}{(y-3)} \cdot \frac{x(y-3)(y+3)}{(x-3)} = \frac{(x-3)\cdot x \cdot (y-3)\cdot(y+3)}{(y-3)\cdot(x-3)}$

$= \frac{x(y+3)}{1} = \underline{\underline{x(y+3)}}$

Order of operations

$2 + 3 \cdot 4 = 14$

$(2+3) \cdot 4 = 20$

$-3^2 = (-1) \cdot 3 \cdot 3 = -9$

$(-3)^2 = (-3) \cdot (-3) = 9$

$-3 \cdot 4 = -12$

Alg/Chain

Roots / Powers

$\sqrt{5} = 5^{0,5} = 5^{\frac{1}{2}}$

$\sqrt{5} \cdot \sqrt{5} \overset{?}{=} 5^{0,5} \cdot 5^{0.5} = 5^{0,5+0,5} = 5^1 = 5$ - ok

$\sqrt[3]{5} = 5^{\frac{1}{3}}$ so $\left(\sqrt[3]{5}\right)^6 = \left(5^{\frac{1}{3}}\right)^6 = 5^{\frac{1}{3}\cdot 6} = 5^2$

Moreover: $5^{-1} = \frac{1}{5}$ and $5^{-2} = \frac{1}{5^2}$

Pattern If m, n integers, $n > 0$ and $a > 0$ (pos. number), then

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Probl 61 $\dfrac{\sqrt{1.03}^{10}}{1.03^4} = \dfrac{\left(1.03^{\frac{1}{2}}\right)^{10}}{1.03^4} = \dfrac{1.03^5}{1.03^4}$

$= 1.03^{5-4} = \underline{1.03}$

Problem Calculate $1.11^{\sqrt{2}}$ on your calculator
(answer: 1.159 035...)

Answer: 1.11 y^x 2 $\sqrt{x}$ =

Same base: $2^{1.5} \cdot 2^{3.8} = 2^{1.5+3.8} = 2^{5.3}$

Same exponent: $2^4 \cdot 3^4 = 2\cdot2\cdot2\cdot2 \cdot 3\cdot3\cdot3\cdot3$

$= 2\cdot3\cdot2\cdot3\cdot2\cdot3\cdot2\cdot3$

Ex: $\sqrt{2}\cdot\sqrt{3} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}$

$= (2\cdot3)^{\frac{1}{2}} = \sqrt{6}$

$= (2\cdot3)^4 = 6^4$

Pattern $a^r \cdot b^r = (ab)^r$

Problem calc. 1.12^{-1} on your calc.

Solution 1 1.12 [y^x] 1 [+/-] [=]

Solution 2 1.12 [1/x] (reason: $1.12^{-1} = \frac{1}{1.12}$)

Absolute value

Ex $\sqrt{(-3)^2} = \sqrt{(-3)\cdot(-3)} = \sqrt{9} = 3 = -(-3) = |-3|$

so $\sqrt{x^2} = |x|$

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Start: 11.00

2. Relative change and rate of change

$$\text{Relative change} = \frac{\text{new value} - \text{old value}}{\text{old value}}$$

Recall $\% = \frac{1}{100} = 0.01$

$3\% = 3 \cdot \frac{1}{100} = 0.03$

Ex Kåre's hourly wage increased from 163 kr to 181 kr. The relative change was

$$\frac{181\text{ kr} - 163\text{ kr}}{163\text{ kr}} = \frac{18}{163} = 11.0\%$$

Rate of change = 1 + relative change

Ex The rate of change in Kåre's hourly wage is $1 + 0.11 = 1.11$

Prob Last year Kåre earned 54000 kr with 163 kr/hour. If he works as much this year as last year, how much will he earn? (with the new wage)

Solution $54000 \cdot 1.11 = \underline{59940}$

3. Interest

Ex You deposit 40000 into an account earning 2.3 % annual interest
Interest is added after each year
(annual compounding of interest)
After a year the balance (what's in the account) is

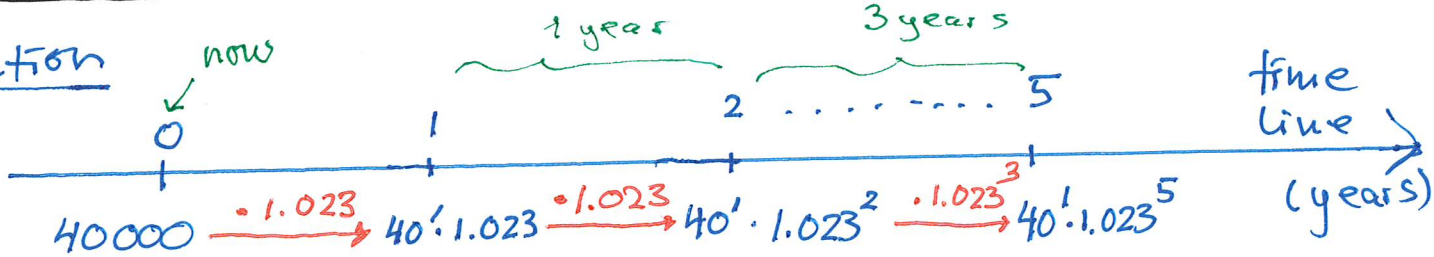
$$40000 + 40000 \cdot 2.3\%$$

$$= 40000 \cdot (1 + 2.3\%) = \underline{40920.00}$$

$1 + 2.3\%$: rate of change, the growth factor

Problem What is the balance after 5 years?

Solution



$$\underline{40000 \cdot 1.023^5 = 44816.52}$$

Ex You deposit 40000 with 2.3% nominal annual interest, but with quaterly compounding of interest. The growth factor (rate of change) for one period (= 3 months) is

$$1 + \frac{2.3\%}{4} = 1 + 0.575\% = 1.00575$$

0.575%: interest for one period

After 1 year the balance is

$$40000 \cdot 1.00575^4 = 40927.96$$

The annual growth factor (rate of change) is $1.00575^4 = 1.023199$

The effective (annual) interest is

$$1.00575^4 - 1 = 0.023199 = 2.3199\%$$

Pattern

$$B = B_0 \cdot \left(1 + \frac{r}{n}\right)^m$$

B: balance after m periods

B_0: deposit (principal)

r: nominal interest

n: number of interest periods

m: number of periods

4. Present value

Let K_0 be some investment/deposit/payment today. The future value K_n of K_0 in years (or more generally, n periods) with interest r is

$$K_n = K_0 \cdot (1+r)^n$$

The opposite: Suppose K_n will be paid n years (periods) from now, with period interest r. Then the present value K_0 of K_n is given as

$$K_0 = \frac{K_n}{(1+r)^n}$$

Problem 30 mill. is paid 5 years from now with 8% (annual) interest. Determine the present value.

Solution $K_0 = \dfrac{30\,\text{mill}}{1.08^5} = 20.42\,\text{mill}$