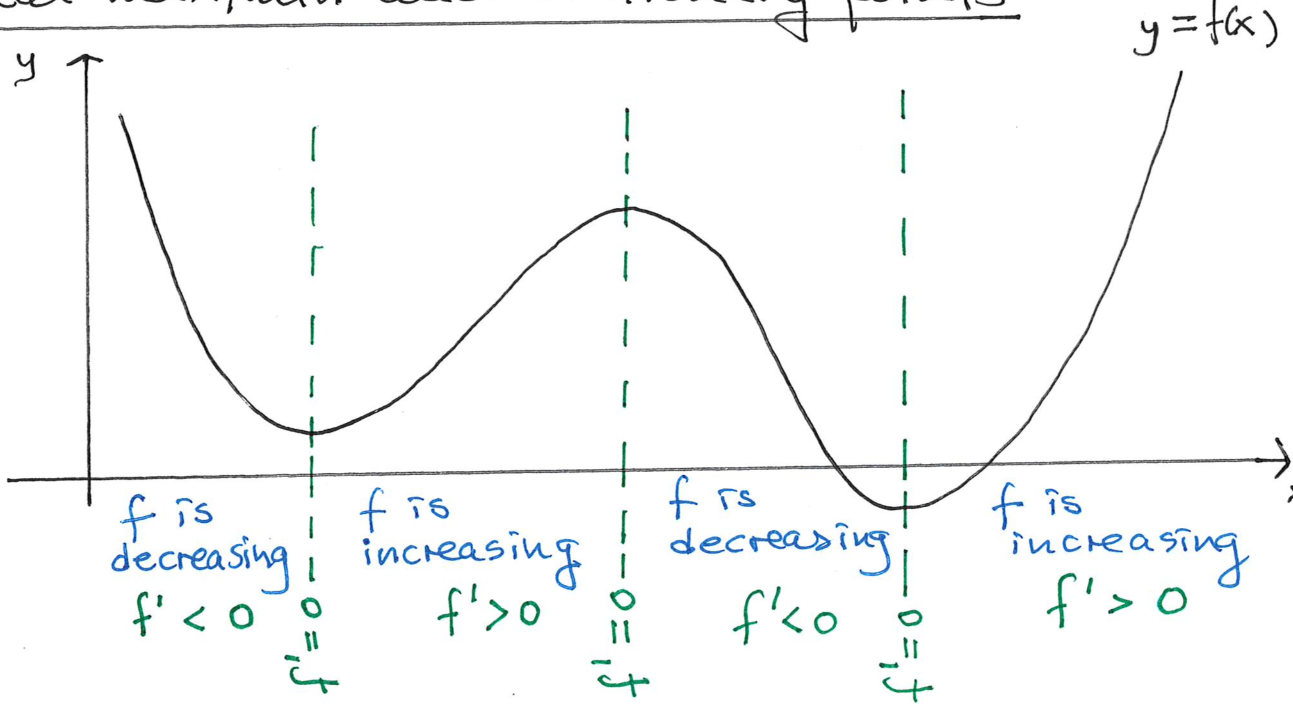
EBA 1180, lecture 18, 25 Oct. 2023, Runar Ile

Plan:
1. Local max/min and stationary points
2. Global max/min
3. The mean value theorem

1. Local max/min and stationary points



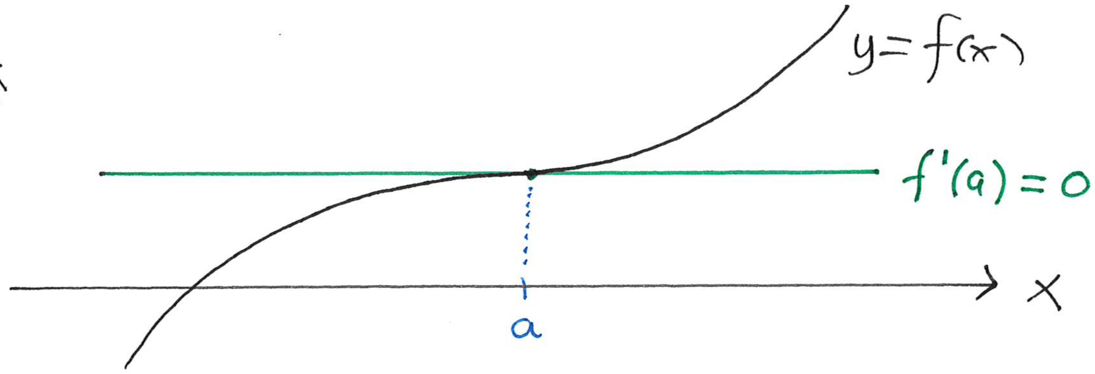
When $f'(x)$ is positive, $f(x)$ is increasing
When $f'(x)$ is negative, $f(x)$ is decreasing

Important conclusion: The sign diagram of $f'(x)$ determines where $f(x)$ is increasing and decreasing

If $x = a$ is a local minimum point, then
$f'(a) = 0$ and $f'(x)$ changes sign from $-$ to $+$

If $x = a$ is a local maximum point, then
$f'(a) = 0$ and $f'(x)$ changes sign from $+$ to $-$

Ex



Here $x=a$ is neither a loc. max. point nor a loc. min. point.
It is a terrace point.

Definition: If $f'(a)=0$ then $x=a$ is a stationary point.

Ex $f(x) = x^3 - 6x^2 + 5$. We find the stationary points – we simply solve the eq. $f'(x)=0$

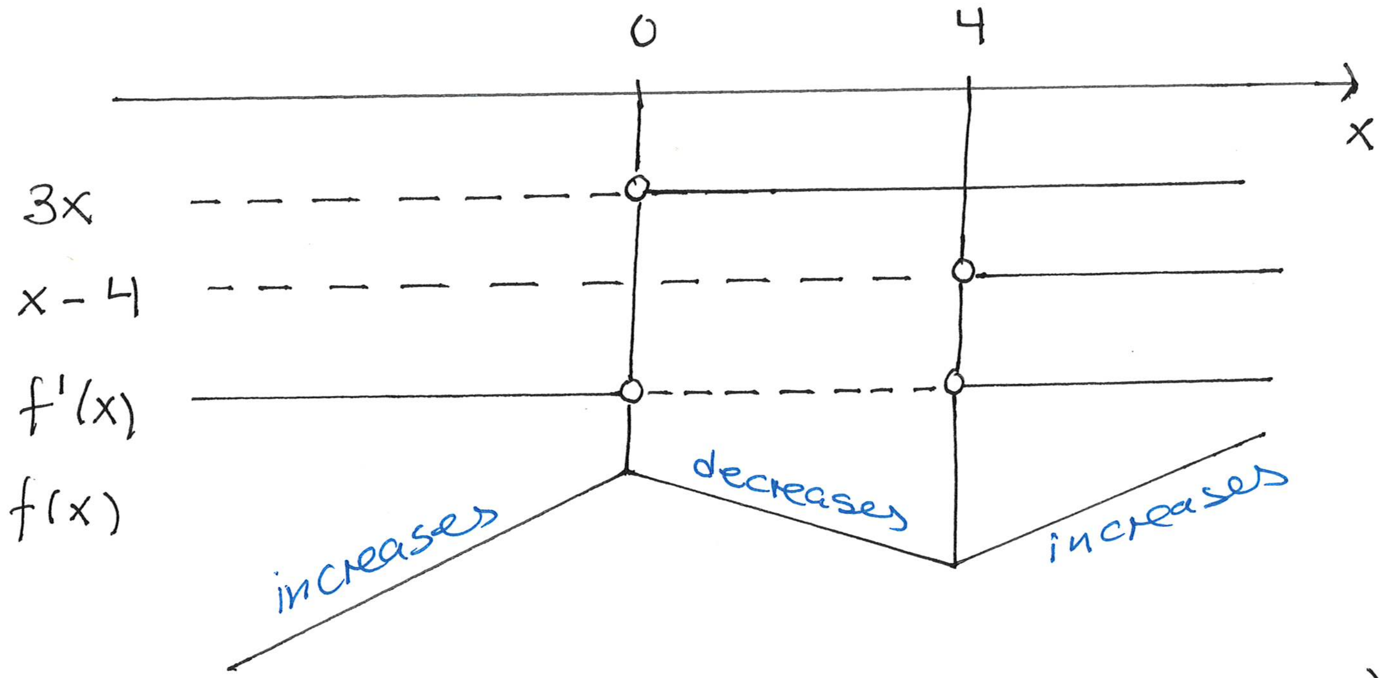
First we find $f'(x) = 3x^2 - 6\cdot 2x + 0$

$= 3x^2 - 12x$

$= 3x(x-4)$

So $f'(x)=0$ has solutions $x=0$ and $x=4$

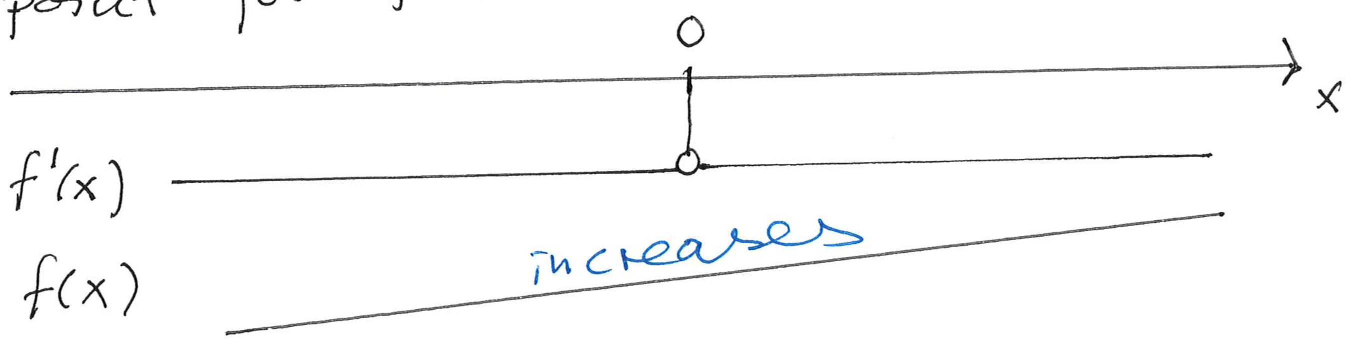
Where is $f(x)$ increasing/decreasing?
– we determine the sign of $f'(x)$ by a sign diagram.



$f(x)$ is strictly increasing for $x \leq 0$ (so $x \in \langle \leftarrow, 0]$)
$f(x)$ is strictly decreasing for $0 \leq x \leq 4$ (so $x \in [0, 4]$)
$f(x)$ is strictly increasing for $x \geq 4$ (so $x \in [4, \rightarrow\rangle$)

Then $x = 0$ is a local maximum point
and $x = 4$ is a local minimum point

<u>Ex</u> $f(x) = x^3 + 1$
so $f'(x) = 3x^2$ and $x = 0$ is the only stationary point for $f(x)$.



<u>Conclusion</u> $f(x)$ is strictly increasing for all numbers on the number line
$(x \in \mathbb{R})$

Start : 11.00

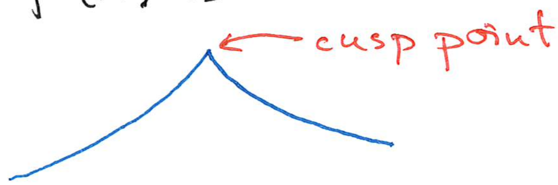
2. Global max/min

The extreme value theorem If $f(x)$ is a continuous function (graph is one snake) on the interval $D_f = [a, b]$ then $f(x)$ has a global maximum and a global minimum.

Possible max/min points:

(*) stationary points (solve $f'(x) = 0$)

(*) cusp points (where $f'(x)$ is not defined)



(*) end points ($x=a, x=b$)

Ex $f(x) = x^3 - 6x^2 + 5$ and $D_f = [-1, 7]$

Find max/min. of $f(x)$.

Solution

(*) stationary points: $f'(x) = 3x^2 - 12x = 0$ gives $\underline{x=0}$, $\underline{x=4}$

(*) cusp points: none

(*) end points: $\underline{x=-1}$, $\underline{x=7}$

These four points are my candidate points for max/min.

Calculate:

$f(-1) = -2$ $\quad f(4) = -27$

$f(0) = 5$ $\quad f(7) = 54$

So $x = 4$ gives the glob. minimum $f(4) = \underline{\underline{-27}}$

and $x = 7$ gives the glob. maximum $f(7) = \underline{\underline{54}}$

EX $f(x) = 12 - x$ with $D_f = [3, 10]$

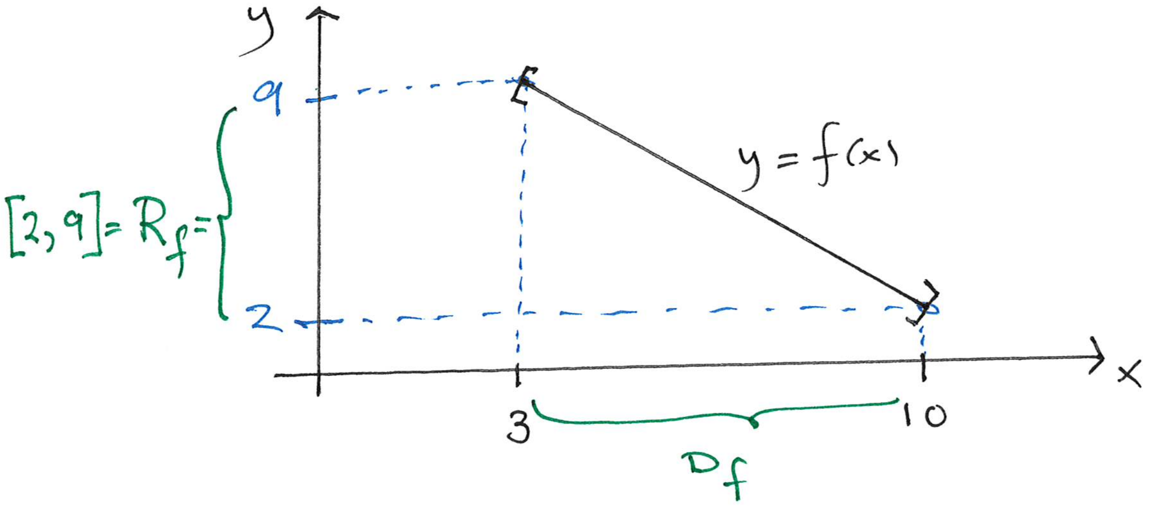
(*) $f'(x) = -1 \neq 0$: no stationary points

(*) no cusp points ($f'(x)$ defined everywhere)

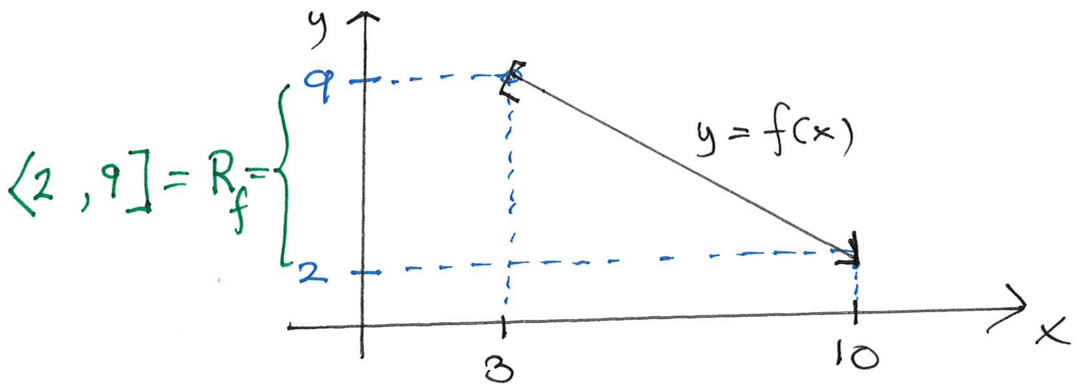
(*) end points: $x = 3$ is a max. point

$x = 10$ is a min. point

($f(x)$ is decreasing in the whole domain)



EX $f(x) = 12 - x$ with $D_f = [3, 10\rangle$

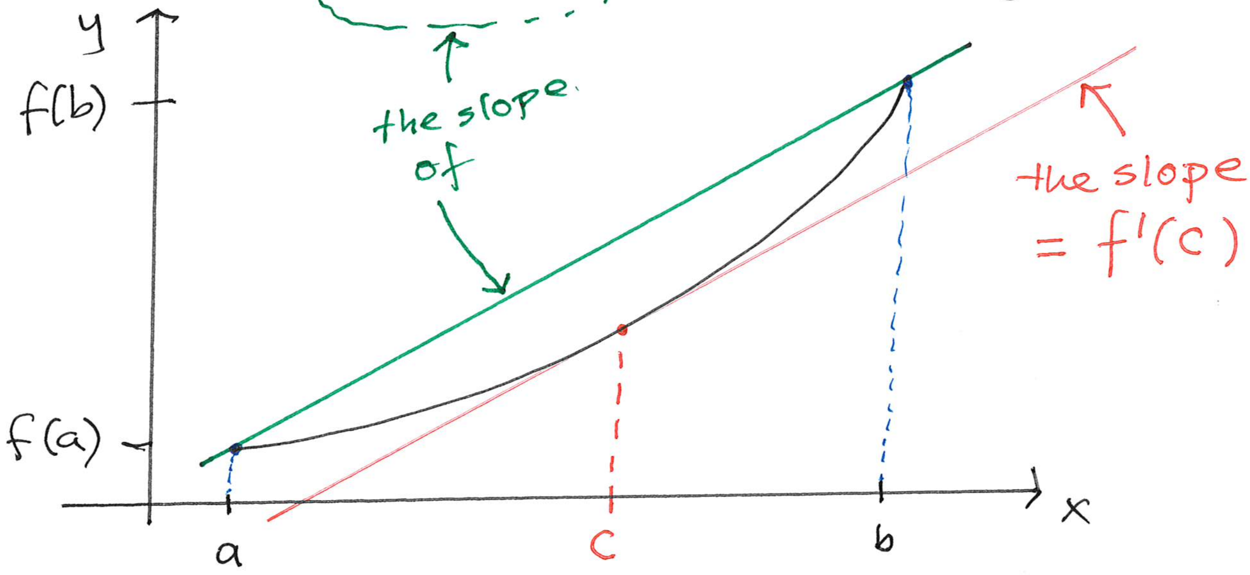


So $x = 3$ is still the max. point and $f(3) = 9$ is the maximum value, but there are no minimum points or minimum values.

3. The mean value theorem

If $f(x)$ is continuous (connected graph) in the interval $[a, b]$ and differentiable (no cusps), then there is a number c between a and b $(a < c < b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{\text{tot. change in } y}{\text{tot. change in } x}$$



green and red line are parallell (same slope)

<u>EX</u> $f(x) = e^x + x^2$. Then $f(0) = e^0 + 0^2 = 1$ and $f(1) = e^1 + 1^2 = e+1$ (so $a=0$, $b=1$)

By the mean value thm. there is a number c between 0 and 1 such that

$$f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{e+1-1}{1} = e$$

<u>Note</u> $f'(x) = e^x + 2x$ (easy), but we <u>cannot</u> find an exact solution to the eq. $e^x + 2x = e$