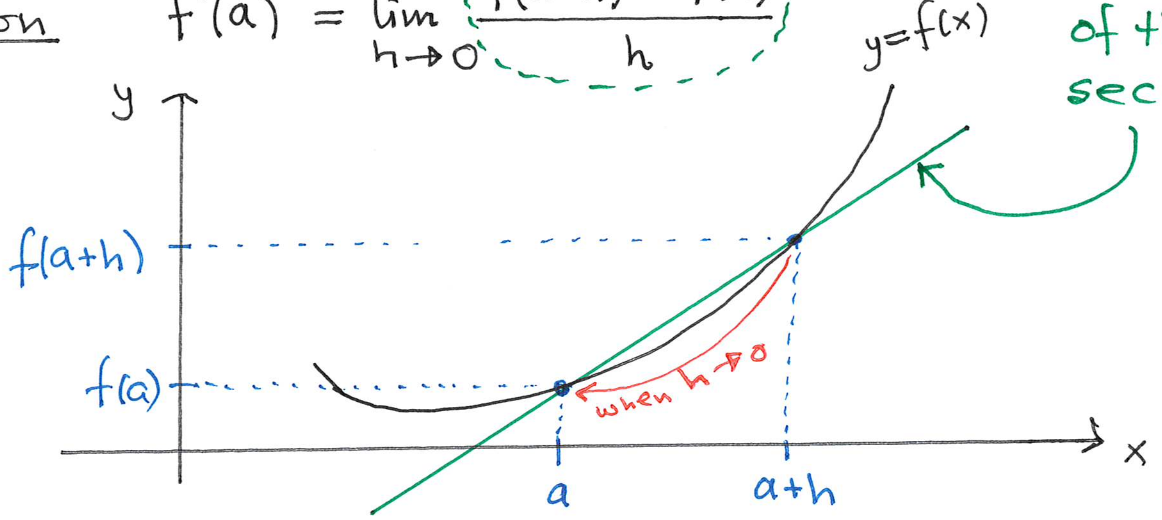
EBA 1180, lecture 17, 24 Oct. 2023, Runar Ile

Plan: Repetition of differentiation
1. Definition, slopes and graphs
2. The natural logarithm
3. Rules of differentiation

1. Definition, slopes and graphs

<u>Definition</u> $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

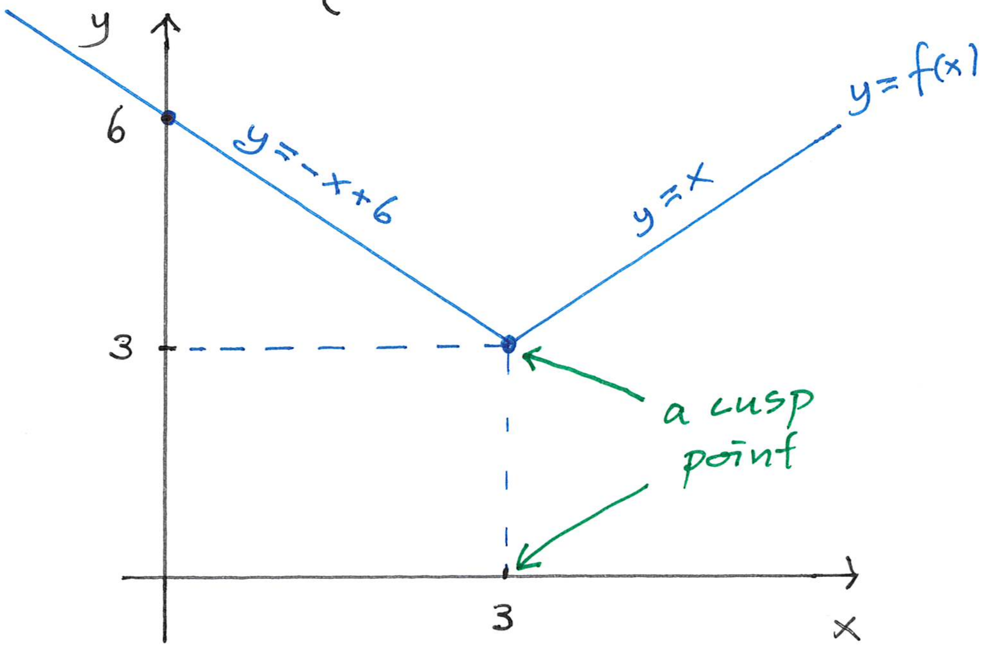
the slope of the secant



<u>Note</u> The derivative does <u>not</u> <u>always</u> exist!

<u>Ex</u> $f(x) = |x-3| + 3 = \begin{cases} -(x-3)+3 & \text{if } x < 3 \\ x-3+3 & \text{if } x \geq 3 \end{cases}$

$= \begin{cases} -x+6 & \text{if } x < 3 \\ x & \text{if } x \geq 3 \end{cases}$

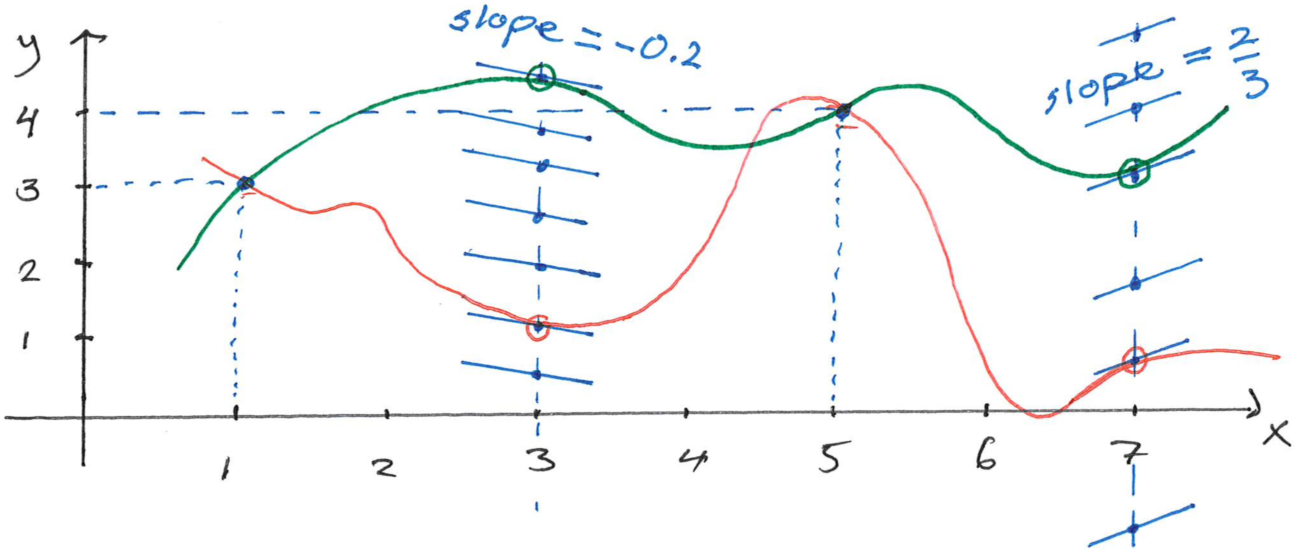


Here $f'(x) = \begin{cases} -1 & \text{if } x < 3 \\ 1 & \text{if } x > 3 \end{cases}$

But for $x = 3$ there is <u>no</u> tangent: hence $f'(3)$ does <u>not</u> exist.

Probl. 1d from last week sketch two graphs.

$f(1)=3$, $f'(3)=-0.2$, $f(5)=4$, $f'(7)=\frac{2}{3}$



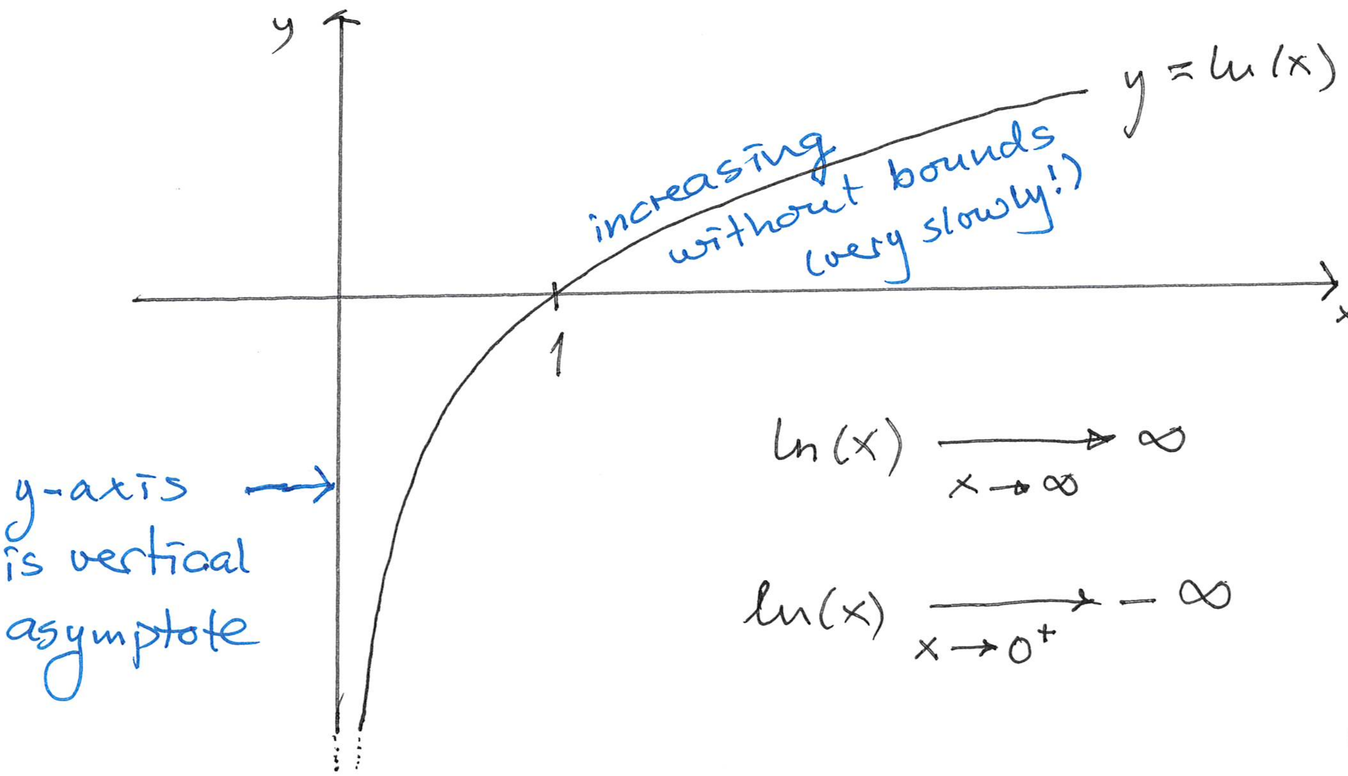
2. The natural logarithm

$\ln(x)$ is the inverse function of e^x

so $\ln(e^x) = x$ and $e^{\ln(x)} = x$

Domain of definition for $\ln(x)$ is the range of e^x : all positive numbers

The range of $\ln(x)$ is the domain of e^x : the whole number line.



$\ln(x) \xrightarrow[x\to\infty]{} \infty$

$\ln(x) \xrightarrow[x\to 0^+]{} -\infty$

Ex $\ln(\sqrt[10]{e}) = \ln(e^{\frac{1}{10}}) = \frac{1}{10}\cdot \ln(e) = \frac{1}{10}\cdot 1$

$= \underline{\underline{\frac{1}{10}}}$

$\ln(3e) = \ln(3) + \ln(e) = \underline{\underline{\ln(3)+1}}$

$e^{2\ln(5)} = e^{\ln(5^2)} = 5^2 = \underline{\underline{25}}$

$= (e^{\ln(5)})^2 = 5^2$

$e^{\ln(2)+\ln(3)} = e^{\ln(2)} \cdot e^{\ln(3)} = 2\cdot 3 = \underline{\underline{6}}$

Note: $\ln(2+3) \neq \ln(2) + \ln(3)$

$\ln(2+3) = 1.6094$

$\ln(2)+\ln(3) = 0.6931 + 1.0986$

$= 1.7918$

Ex $\ln(5x) = \ln(5) + \ln(x)$

$\ln(x^{10}) = 10 \cdot \ln(x)$

$\ln\left(\frac{3}{x-1}\right) = \ln(3) - \ln(x-1)$

Start: 11.01

3. Rules of differentiation

Product rule $\left[g(x)\cdot h(x)\right]' = g'(x)\cdot h(x) + g(x)\cdot h'(x)$

Ex $\left[(x^2+1)\cdot e^x\right]' = 2x\cdot \underline{e^x} + (x^2+1)\cdot \underline{e^x}$ common factor

$= \underline{\underline{(x^2+2x+1)\cdot e^x}}$ – zero? $x=-1$

Problem $\left[\sqrt{x}\cdot \ln(x)\right]' = (\underbrace{\sqrt{x}}_{= x^{\frac{1}{2}}})'\cdot \ln(x) + \underbrace{\sqrt{x}}_{= x^{\frac{1}{2}}}\cdot\left[\ln(x)\right]'$

$= \frac{1}{2}x^{\frac{1}{2}-1}\cdot \ln(x) + x^{\frac{1}{2}}\cdot x^{-1}$

$= \frac{1}{2}\underline{x^{-\frac{1}{2}}}\cdot \ln(x) + \underline{x^{-\frac{1}{2}}}$ – zero?

$= x^{-\frac{1}{2}}\cdot\left(\frac{1}{2}\cdot \ln(x) + 1\right) \quad \Big| \cdot \frac{2}{2} = 1$

$= \frac{\ln(x)+2}{2\sqrt{x}}$

zero: $\ln(x)+2=0$

$\ln(x) = -2$

$x = e^{\ln(x)} = e^{-2}$

pos?: $x > e^{-2}$

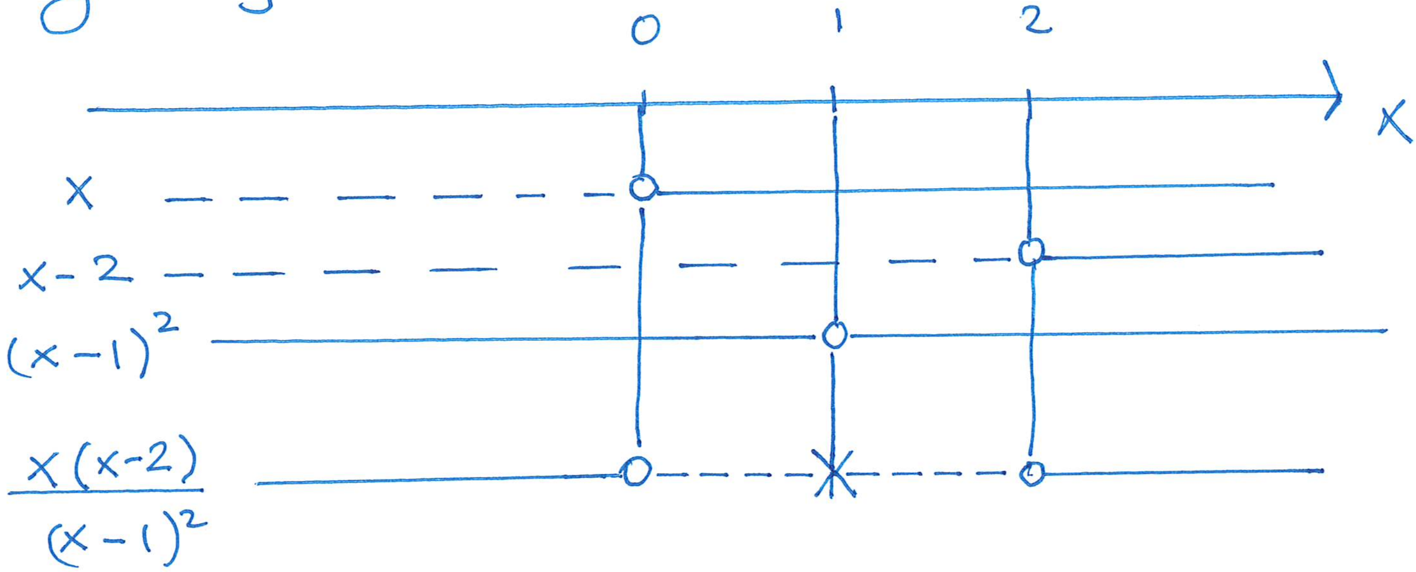
Quotient rule

$$\left[\frac{g(x)}{h(x)}\right]' = \frac{g'(x)\cdot h(x) - g(x)\cdot h'(x)}{\left[h(x)\right]^2}$$

Ex $\left[\frac{x^2}{x-1}\right]' = \frac{2x\cdot(x-1) - x^2\cdot 1}{(x-1)^2} = \frac{\underline{x^2} - \underline{2x}}{(x-1)^2}$

$= \underline{\underline{\frac{x(x-2)}{(x-1)^2}}}$ – pos? $x>2$ or $x<0$

Sign diagram



Ex $\left[\dfrac{\ln(x)}{x}\right]' = \dfrac{\frac{1}{x}\cdot x - \ln(x)\cdot 1}{x^2}$

$= \dfrac{1-\ln(x)}{x^2}$

zero: $x = e$

pos: $0 < x < e$

Chain rule $[g(u(x))]' = g'(u)\cdot u'(x)$ where $u = u(x)$

Ex $\left[e^{x^2+3x}\right]' = e^u \cdot (2x+3) = \underline{\underline{(2x+3)\cdot e^{x^2+3x}}}$

$u(x) = x^2+3x$ and $g(u) = e^u$

$u'(x) = 2x+3$ $\quad g'(u) = e^u$

Probl $[\ln(x^2+5)]' = \dfrac{1}{u}\cdot 2x = \underline{\underline{\dfrac{2x}{x^2+5}}}$

$u(x) = x^2+5$ and $g(u) = \ln(u)$

$u'(x) = 2x$ $\quad g'(u) = \dfrac{1}{u}$

Ex $\left[\ln\left(\frac{3x}{x-1}\right)\right]' = \left[\ln(3x) - \ln(x-1)\right]'$

$= \left[\ln(3) + \ln(x) - \ln(x-1)\right]'$

$= 0 + \frac{1}{x} - \frac{1}{x-1} = \frac{x-1-x}{x(x-1)} = \underline{\underline{\frac{-1}{x(x-1)}}}$

$u(x) = x-1$ and $g(u) = \ln(u)$

$u'(x) = 1 \qquad g'(u) = \frac{1}{u}$