EBA 1180, lecture 11, 3 Oct. 2023, Runar Ile

Plan 1. Second degree polynomial functions with problem 5a-e, 7a and 8b.

2. Revenue, cost and profit with problem 9.

1. Second degree polynomial functions

5a) Because we have two easy-to-read zeros, we use the form $f(x) = a(x-\underbrace{r_1)(x-r_2}_{\text{roots}}) = a(x-2)(x-5)$

To find a, note that

$f(0)=5$ so $a\cdot(0-2)\cdot(0-5) = 5$

$a \cdot 10 = 5$

$a = \frac{5}{10} = \frac{1}{2} = 0.5$

and so $\underline{\underline{f(x) = \frac{1}{2}(x-2)(x-5)}}$

5b) $x=2$ is the larger root,

$f(-1) = 6 = f(0)$, so $x = -\frac{1}{2}$ is the symmetry axis (in middle between -1 and 0)

So the smaller root has to be

$x = -\frac{1}{2} - 2,5 = -3$.

Hence $f(x) = a(x-2)(x+3)$

To find a, note $f(0) = 6$, so

$a\cdot(0-2)(0+3) = 6$

$a\cdot(-6) = 6$

$a = \frac{6}{-6} = -1$

and $\underline{\underline{f(x) = -(x-2)(x+3)}}$

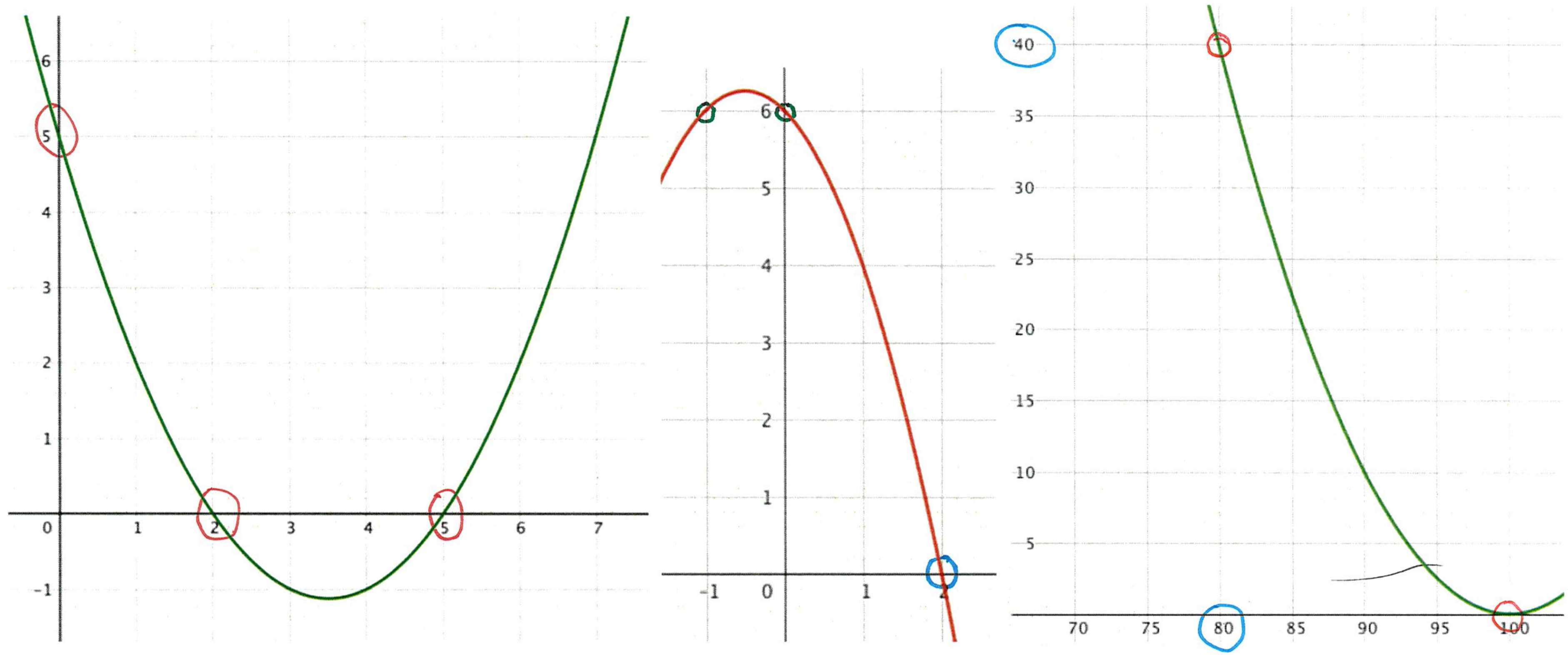


Figure 2: Parabolas (a-c)

5c) We see that $x=100$ is a double root

$f(x) = a(x-100)^2$.

Since $(80, 40)$ is a point on the graph,

$f(80) = 40$, so $a\cdot(80-100)^2 = 40$

$$a\cdot(-20)^2 = 40$$

$$a\cdot 400 = 40$$

$$a = \frac{40}{400} = \frac{1}{10} = 0.1$$

and so $\underline{\underline{f(x) = \frac{1}{10}(x-100)^2}}$

This is the std. form $f(x) = a(x-s)^2 + d$

with $a = \frac{1}{10}$, $s = 100$, $d = 0$.

5d) We observe the symmetry axis $x = 1$

and the maximum value $y = -1$

Then $f(x) = a(x-s)^2 + d$

$$= a(x-1)^2 - 1$$

To find, note $f(0) = -2$, we get

$$a\cdot(0-1)^2 - 1 = -2$$

$$a - 1 = -2$$

$$a = -2+1 = -1.$$

and $\underline{\underline{f(x) = -(x-1)^2 - 1}}$

5e) The symmetry axis is $x = -3$

The minimum value is $y = 4.25$

(in the middle between 4 and 4.5)

so $f(x) = a(x+3)^2 + 4.25$

From $f(-2) = 4.5$ we get

$$a \cdot (-2+3)^2 + 4.25 = 4.50$$

$$a + 4.25 = 4.50$$

$$a = 0.25$$

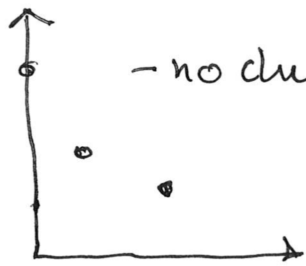
and $\underline{\underline{f(x) = 0.25(x+3)^2 + 4.25}}$

7a) Three points on the graph: $P = (0, 7)$

No extra ('good') info, $Q = (1, 4)$

so use the form $R = (2, 3)$

$f(x) = ax^2 + bx + c$



- no clue.

P: $f(0) = 7$ gives $\underline{c = 7}$.

Q: $f(1) = a \cdot 1^2 + b \cdot 1 + 7 = 4$

$$a + b = -3 \quad (1)$$

R: $f(2) = a \cdot 2^2 + b \cdot 2 + 7 = 3$

$$4a + 2b = -4 \quad (2)$$

Solve this system of equations

Multiply (1) on both sides by 4 and get

$$4a + 4b = -12$$

subtract (2): $\quad 4a + 2b = -4$

$$0a + 2b = -8$$

so $\quad b = \frac{-8}{2} = \underline{-4}$

From (1) we get $\quad a = -3-(-4) = \underline{1}$

so $\quad \underline{\underline{f(x) = x^2 - 4x + 7}}$

8b) $f(x) = 3x^2 + 36x + 110$

How to complete the square?

Note that $\quad 3x^2 + 36x = 3(x^2 + 12x)$

Complete the sq of $\quad x^2 + 12x$:

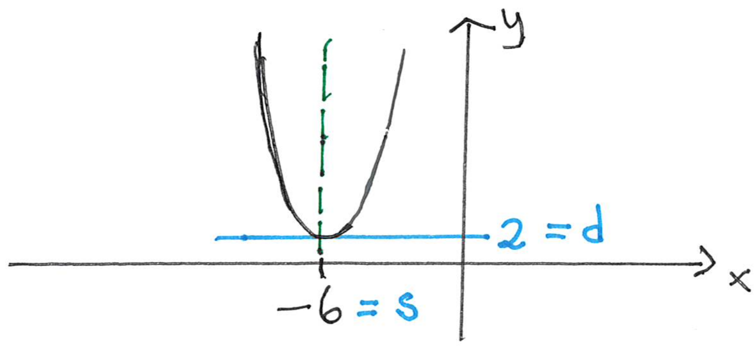
$$x^2 + 12x = (x+6)^2 - 36$$

so $\quad f(x) = 3\left[(x+6)^2 - 36\right] + 110$

$$= 3(x+6)^2 - 108 + 110$$

$$= \underline{\underline{3(x+6)^2 + 2}}$$

so $\quad \underline{\underline{a = 3}}, \quad \underline{\underline{s = -6}}, \quad \underline{\underline{d = 2}}$



Start: 11.02

Summary (second degree functions)

3 standard forms:

A) If we know the roots: $f(x) = a(x-r_1)(x-r_2)$

B) If we know the symmetry axis: $x = s$

and the max/min value: $y = d$

then $f(x) = a(x-s)^2 + d$

C) Other cases: $f(x) = ax^2 + bx + c$

(but we can always use B).

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2. Revenue, cost and profit

x = number of units produced and sold

p = unit price, so revenue $R(x) = p \cdot x$

Determine p such that the profit is positive when $x > 300$.

a) The cost function is $C(x) = 2100 + 5x$.

The profit function is $P(x) = R(x) - C(x)$

$= px - (2100 + 5x) = (p-5)x - 2100$

By assumption the inequality $P(x) > 0$ should have solution set $x > 300$.

We solve the inequality

$$P(x) > 0 \quad , \quad \text{that is}$$

$$(p-5)x - 2100 > 0 \quad \Big| + 2100$$

$$(p-5)x > 2100 \quad \Big| : (p-5)$$

Two cases :

<u>$p-5 < 0$</u> : $\quad x < \dfrac{2100}{p-5}$ which is a negative number

But the number of units produced and sold cannot be a neg. number – so no solutions in this case

<u>$p-5 > 0$</u> : $\quad x > \dfrac{2100}{p-5}$

and this solution set is supposed to equal $x > 300$ so

$$\frac{2100}{p-5} = 300 .$$ We solve this eq. for p .

that is $p-5 = \dfrac{2100}{300} = 7$

so $p = 7+5 = \underline{\underline{12}}$

b) The cost function is $C(x) = 4500 - 5x + 0.01x^2$ with $x \in [0, 1000]$.

Then $P(x) = px - (4500 - 5x + 0.01x^2)$

resolve and collect terms

$$= -0.01x^2 + (p+5)x - 4500$$

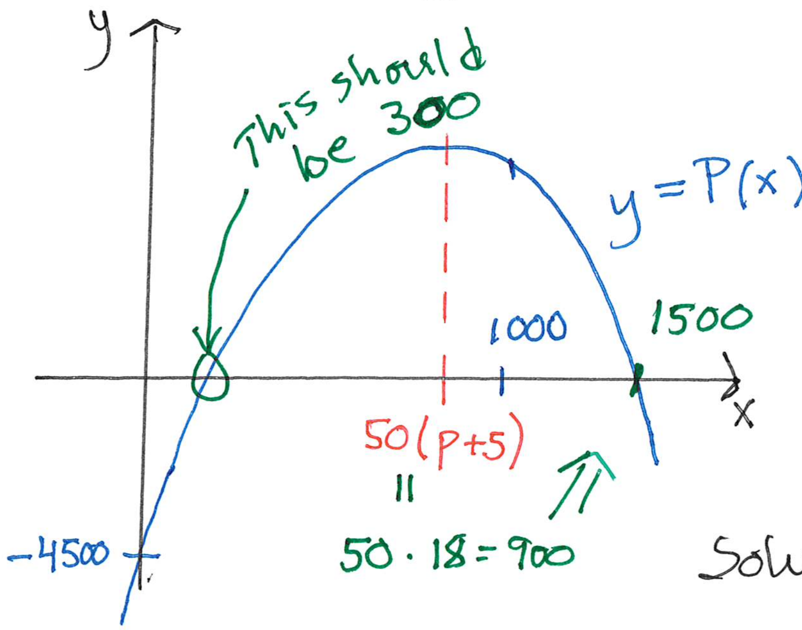
$$= -0.01\left(x^2 - 100(p+5)x\right) - 4500$$

:2

$$= -0.01\left(\left[x - 50(p+5)\right]^2 - 50^2(p+5)^2\right) - 4500$$

square

$$= -0.01\left[x - 50(p+5)\right]^2 + 25(p+5)^2 - 4500$$



Need to find the value of p which makes the smaller root of P(x) equal to 300

Solve eq. $P(300) = 0$ for p.

that is $-0.01 \cdot 300^2 + (p+5) \cdot 300 - 4500 = 0$

$$(p+5) \cdot 300 = 4500 + 900 = 5400$$

so $p + 5 = 5400/300 = 18$.

so $p = 18 - 5 = \underline{\underline{13}}$