

**EBA1180 Mathematics for Data Science  
autumn 2022  
Exercises**

*... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.*

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**Lecture 8 and 9**

**Sec. 4.7, 3.4-5, 2.6:**

**Polynomial division. Factorisation. Rational and radical equations. Inequalities.**

Here are recommended exercises from the textbook [SHSC].

Section 4.7 exercise 1-6

Section 3.4 exercise 1, 2a, b, d

Section 3.5 exercise 2-4

Section 2.6 exercise 1-3, 5

**Problems for the exercise session  
Wednesday 21 Sept. from 12 in B2-065**

**Problem 1** Perform the polynomial division with remainder.

- a)  $(x^2 + 4x - 21) : (x - 3)$       b)  $(x^2 + 4x - 21) : (x - 4)$       c)  $(x^3 + x^2 - 23x + 42) : (x + 6)$   
d)  $(x^3 + x^2 - 23x + 42) : (x + 1)$       e)  $(x^4 + x^2 + 1) : (x^2 - x + 1)$       f)  $(x^2 + 3x - 7) : (x - a)$

**Problem 2** Factorise the polynomial into factors of the least possible degree (by guessing on a zero and performing polynomial division).

- a)  $x^2 + 4x - 221$       b)  $x^3 + 6x^2 - x - 30$       c)  $x^3 - 3x^2 + 5x - 15$   
d)  $x^4 + 10x^3 + 35x^2 + 50x + 24$

**Problem 3** Solve the equations.

- a)  $\frac{5x + 1}{x^2 + x + 1} = -2$       b)  $\frac{x - 1}{x^2 + x + 1} = 1$       c)  $\frac{1}{1 + \frac{1}{x}} = \frac{1}{x}$       d)  $\frac{(x - 1)(x - 3)}{(x - 2)(x - 4)} = 2$

**Problem 4** Determine the values of  $a$  such that the equation has solutions.

- a)  $x^2 + 2ax + 9 = 0$       b)  $\frac{1}{x + a} = \frac{2}{2x + 3}$       c)  $\frac{(x - 1)(x - 3)}{(x - 2)(x - 4)} = a$

**Problem 5** Solve the equations.

a)  $\sqrt{2x+3} = x+2$

b)  $\sqrt{4x+1} = x-1$

c)  $\sqrt{x+2} + \sqrt{x-3} = 5$

d)  $\sqrt{2x+1} - \sqrt{x+4} = 1$

e)  $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = 2$

f)  $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = -1$

**Problem 6** Determine the values of  $a$  such that the equation  $\frac{1}{\sqrt{x}-1} - \frac{1}{\sqrt{x}+1} = a$  has solutions.  
(Note problem 5e and 5f.)

**Problem 7** Solve the inequalities.

a)  $2x+3 \leq 5x+2$

b)  $-4x+1 \geq x-1$

c)  $x+2 < 3+5x$

d)  $(x-5)(x+4) < 0$

e)  $(2x+5)(7-x) \geq 0$

f)  $\frac{x-5}{x+4} \geq 0$

g)  $\frac{(x-2)(x+3)}{(x-5)(x+4)} < 0$

h)  $\frac{-5}{(6-x)(-12-3x)} \geq 0$

i)  $(x-5)(x+4) < 10$

j)  $(2x+5)(7-x) \geq 35$

k)  $\frac{(x-2)(x+3)}{(x-5)(x+4)} < 1$

l)  $\frac{-5}{(6-x)(-12-3x)} \geq \frac{5}{72}$

**Problem 8** Determine  $a$  such that the inequality has solutions.

a)  $x^2 + 6x \leq a$

b)  $(x+a)^2 < a$

## Answers

### Problem 1

- a)  $x + 7$                       b)  $x + 8 + \frac{11}{x-4}$                       c)  $x^2 - 5x + 7$   
d)  $x^2 - 23 + \frac{65}{x+1}$                       e)  $x^2 + x + 1$                       f)  $x + a + 3 + \frac{a^2 + 3a - 7}{x-a}$

### Problem 2

- a)  $(x-13)(x+17)$                       b)  $(x-2)(x+3)(x+5)$                       c)  $(x-3)(x^2+5)$   
d)  $(x+1)(x+2)(x+3)(x+4)$

### Problem 3

- a)  $x = -3, x = -\frac{1}{2}$                       b) no solutions                      c)  $x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$                       d)  $x = 4 \pm \sqrt{3}$

### Problem 4

- a)  $a \leq -3$  or  $a \geq 3$                       b)  $a = \frac{3}{2}$                       c) All values of  $a$  give solutions

### Problem 5

- a)  $x = -1$                       b)  $x = 6$                       c)  $x = 7$   
d)  $x = 12$                       e)  $x = 2$                       f) no solutions

### Problem 6

$a \leq -2$  or  $a > 0$  (compare with 5e and 5f)

### Problem 7

- a)  $x \geq \frac{1}{3}$ , alternative way of writing:  $x \in [\frac{1}{3}, \infty)$   
b)  $x \leq \frac{2}{5}$ , alternative:  $x \in \langle -\infty, \frac{2}{5} ]$   
c)  $x > -\frac{1}{4}$ , alternative:  $x \in \langle -\frac{1}{4}, \infty)$   
d)  $-4 < x < 5$ , alternative:  $x \in \langle -4, 5)$   
e)  $-\frac{5}{2} \leq x \leq 7$ , alternative:  $x \in [-\frac{5}{2}, 7]$   
f)  $x < -4$  or  $x \geq 5$ , alternative:  $x \in \langle \infty, -4) \cup [5, \infty)$   
g)  $-4 < x < -3$  or  $2 < x < 5$ , alternative:  $x \in \langle -4, -3) \cup \langle 2, 5)$   
h)  $-4 < x < 6$ , alternative:  $x \in \langle -4, 6)$   
i)  $-5 < x < 6$ , alternative:  $x \in \langle -5, 6)$   
j)  $0 \leq x \leq \frac{9}{2}$ , alternative:  $x \in [0, \frac{9}{2}]$   
k)  $x < -7$  eller  $-4 < x < 5$ , alternative:  $x \in \langle -\infty, -7) \cup \langle -4, 5)$   
l)  $-4 < x \leq 0$  or  $2 \leq x < 6$ , alternative:  $x \in \langle -4, 0] \cup [2, 6)$

### Problem 8

- a)  $a \geq -9$                       b)  $0 < a$