Exercise session problems

Problem 1.

Consider the subset $D \subseteq \mathbb{R}^2$ given by the inequality $y(x - 2) \leq 3$. Make a sketch of $D = \{(x,y) : y(x - 2) \leq 3\}$, and mark the inner points and the boundary points of D. Is D compact?

Problem 2.

Consider a subset of the plane $\mathbb{R}^2$ given by the following conditions. Determine whether the subset is compact. It is useful to make a sketch of the area.

a) $2x + 3y = 6$ b) $2x + 3y < 6$ c) $2x + 3y \leq 6$ d) $x^2 + y^2 = 4$

e) $x^2 + y^2 \geq 4$ f) $x^2 + y^2 \leq 4$ g) $x^2 - 2x + 4y^2 = 4$ h) $x^2 - 2x + 4y^2 \leq 4$

i) $x^2 - 2x + 4y^2 \geq 4$ j) $xy = 1$ k) $xy \leq 1$ l) $xy \geq 1$

m) $\sqrt{x^2 + y^2} = 3$ n) $\sqrt{x^2 + y^2} \leq 3$ o) $x^2y^2 - x^2 - y^2 + 1 = 0$ p) $x^2y^2 - x^2 - y^2 + 1 = 1$

Problem 3.

State the extreme value theorem. Give examples of a set D in the plane which is closed, but not bounded, as well as a set E in the plane which is bounded, but not closed. Can you find a function $f(x,y)$ which does not have a maximum nor a minimum in D, and a function which does not have a maximum nor a minimum in E?

Problem 4.

Level curves for the functions $f(x,y) = 4x^2 + 9y^2$ and $g(x,y) = x^2y^2 - x^2 - y^2 + 1$ in the area $-4 \leq x,y \leq 4$ are shown in the figures below.
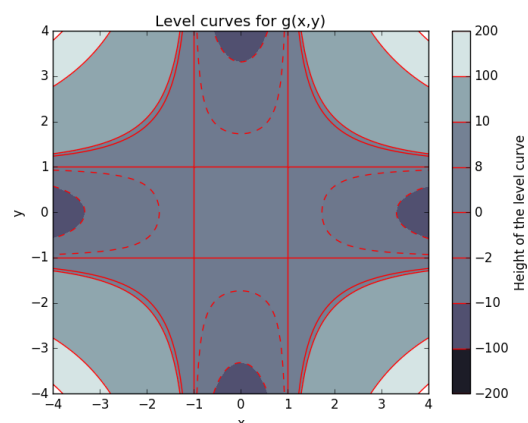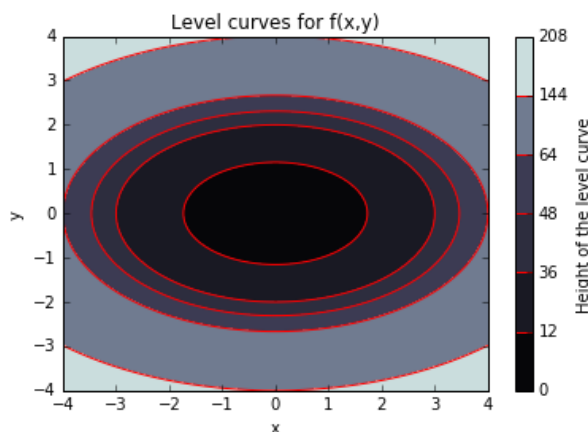
a) Find max / min $f(x,y)$ when $-4 \leq x,y \leq 4$ by using the figure.

b) Find max / min $g(x,y)$ when $-4 \leq x,y \leq 4$ by using the figure.

c) Find max / min $f(x,y)$ when $x^2 + y^2 = 16$ by using the figure.

d) Find max / min $g(x,y)$ when $x = y$ by using the figure.

Problem 5.

Solve the optimization problems:

a) $\max/\min f(x,y) = x^3 - 3xy + y^3$ når $0 \leq x,y \leq 1$ b) $\max/\min f(x,y) = x^3 - 3xy + y^3$ når $0 \leq x,y \leq 2$

c) $\max/\min f(x,y) = e^{xy-x-y}$ når $0 \leq x,y \leq 2$ d) $\max/\min f(x,y) = xy(x^2 - y^2)$ når $-1 \leq x,y \leq 1$

e) $\max/\min f(x,y) = (x^2 - 1)(y^2 - 1)$ når $-1 \leq x,y \leq 1$

Problem 6.

Find the maximum- and minimum value for the optimization problem

$$\max/\min f(x,y) = \sqrt{xy} - x \text{ når } 0 \leq x,y \leq 1$$

Optional: Exercises from the Norwegian textbook

Textbook [E]:   Eriksen, *Matematikk for økonomi og finans*
Exercise book [O]:   Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag*

Exercises:	[E] 7.6.1 - 7.6.2
Solution manual:	See [O] Ch. 7.6

Answers to the exercise session problems

Problem 1.

Boundary points are given by the equation $y(x-2) = 3$, that is points on the graph of $y = 3/(x-2)$ (a hyperbola). Inner points are given by $y(x-2) < 3$, that is points under the hyperbola when $x > 2$, and points over the hyperbola when $x < 2$, as well as all points where $x = 2$. The set D is not compact (closed, but not bounded).

Problem 2.

a) No	b) No	c) No	d) Yes	e) No	f) Yes	g) Yes	h) Yes
i) No	j) No	k) No	l) No	m) Yes	n) Yes	o) No	p) No

Problem 4.

a) $f_{\min} = 0$ in $(0,0)$, and $f_{\max} = 208$ in $(\pm 4, \pm 4)$

b) $f_{\min} = -15$ in $(0, \pm 4)$ and $(\pm 4,0)$, and $f_{\max} = 225$ in $(\pm 4, \pm 4)$

c) $f_{\min} = 64$ in $(\pm 4,0)$, and $f_{\max} = 144$ in $(0, \pm 4)$

d) $f_{\min} = 0$ in $(1,1)$ and $(-1, -1)$, and $f_{\max} = 225$ in $(4,4)$ and $(-4, -4)$

Problem 5.

a) $f_{\max} = 1$, $f_{\min} = -1$	b) $f_{\max} = 8$, $f_{\min} = -1$	c) $f_{\max} = 1$, $f_{\min} = 1/e^2$
d) $f_{\max} = 2\sqrt{3}/9$, $f_{\min} = -2\sqrt{3}/9$	e) $f_{\max} = 1$, $f_{\min} = 0$	

Problem 6.

See the final exam of MET11807 06/2021 Exercise 5: $f_{\max} = 1/4$, $f_{\min} = -1$