EBA 1180, lecture 9, 21 Sept. 2022, Runar Ile

Plan
1. Rational and radical equations
2. Inequalities

1a. Rational equations

A rational equation: $\frac{p(x)}{q(x)} = 0$

where $p(x)$ and $q(x)$ are polynomials.

EX Eq. $\frac{x+1}{(x-1)(x+3)} = 0$ then $x+1=0$

and $(x-1)(x+3) \neq 0$

i.e. $x \neq 1$ and $x \neq -3$

so $\underline{\underline{x=-1}}$

EX (Probl. 10a from last week)

$1 + x + x^2 + \dots + x^{99} = 0$

This is a geometric series with $a_1 = 1$, $k = x$, $n = 100$ which gives

$$1 \cdot \frac{x^{100}-1}{x-1} = 0 \qquad (x \neq 1)$$

then $x^{100} - 1 = 0$ so $x^{100} = 1$ $(x \neq 1)$

so $x = \pm 1^{\frac{1}{100}} = \pm 1$ $(x \neq 1)$

so $\underline{\underline{x = -1}}$

Note: $x=1$ is not a solution since then the LHS $=100$

Ex Eq. $\dfrac{x+1}{(x-1)(x+3)} = 2$

$$\frac{x+1}{(x-1)(x+3)} - 2 = 0$$

multiply -2 with $\dfrac{(x-1)(x+3)}{(x-1)(x+3)} = 1$

Get $\dfrac{x+1 - 2(x-1)(x+3)}{(x-1)(x+3)} = 0$

$$\frac{x+1 - 2(x^2+2x-3)}{(x-1)(x+3)} = 0$$

$$\frac{-2x^2 - 3x + 7}{(x-1)(x+3)} = 0$$

that is $-2x^2 - 3x + 7 = 0$

with $x \neq 1$, $x \neq -3$

which you can solve.

1b. Radical equations

– the unknown is under a root!

Ex $2\sqrt{x+1} = x-2$ $\quad (x \geq -1)$

square both sides

$$4(x+1) = (x-2)^2 = (x-2)(x-2) = x^2-4x+4$$

$$4x+\cancel{4} = x^2-4x+\cancel{4}$$

$$x^2-8x = 0$$

$$x(x-8) = 0 \quad \text{so } \underline{x=0} \text{ or } \underline{x=8}$$

Note Not all of these x-values need to be solutions of the original equation.

We have to test the candidates:

$x = 0$ LHS $2\cdot\sqrt{0+1} = 2\sqrt{1} = 2$ } not equal so $x = 0$ is not a solution
RHS $0 - 2 = -2$

$x = 8$ LHS $2\cdot\sqrt{8+1} = 2\cdot\sqrt{9} = 6$ } - equal! so $x = 8$ is the only solution
RHS $8 - 2 = 6$

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2. Inequalities

$-2 < -1$ read: 'minus two is less than minus one'

$\frac{1}{9} > \frac{1}{12}$ read: 'one ninth is bigger than one twelfth'

Also $\leqslant$ and $\geqslant$

- An inequality is a claim that one expression (number) is less than, bigger than ... another expression (number)
- The solutions of an inequality are those values of x which make the claim true

EX $x - 1 \geqslant 2$ is a claim
- is true if $x = 5$ since $5 - 1 \geqslant 2$ true
- is not true if $x = 2$ since $2 - 1 \geqslant 2$ not true

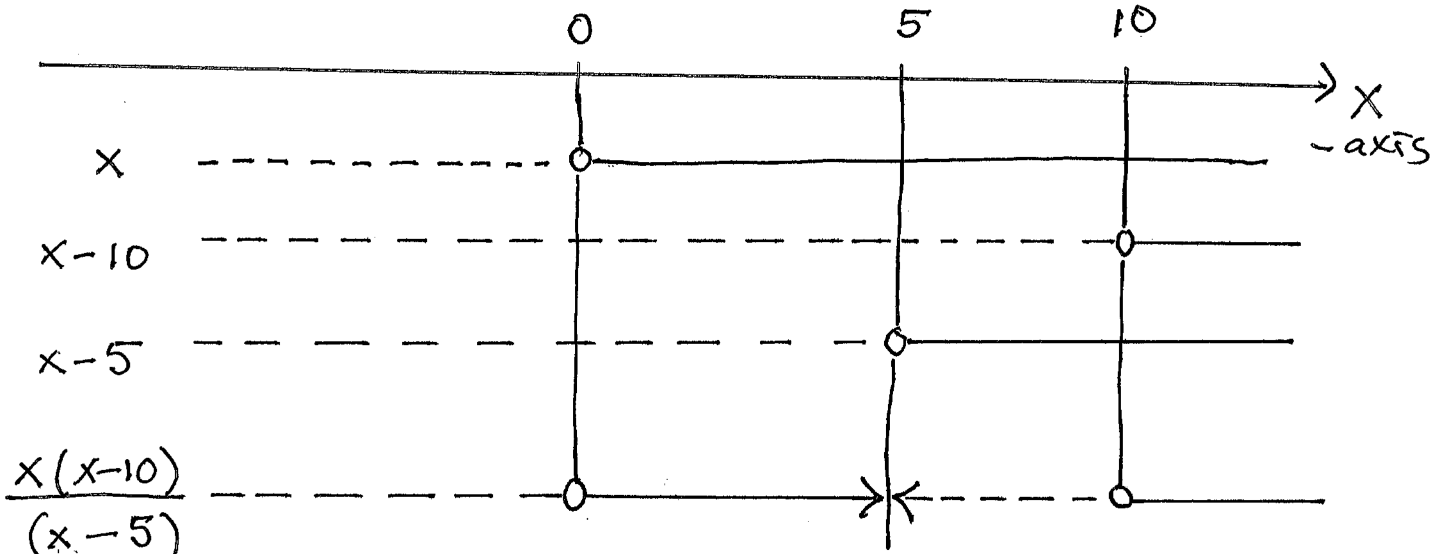
Start: 11.01

The solutions of the inequality are the values of x such that

$x \geqslant 3$ - an infinite set of numbers.

Ex Solve the inequality $\dfrac{x(x-10)}{x-5} \geqslant 0$

Solution Because we have 0 on the RHS and a factorised LHS we can use a sign diagram.



that is $0 \leqslant x < 5$ or $x \geqslant 10$

We also write $x \in [0, 5\rangle$ or $x \in [10, \infty\rangle$

Problem $\dfrac{2x-12}{(x-3)(x+4)} \geqslant 1$ (Course Paper 2020a)