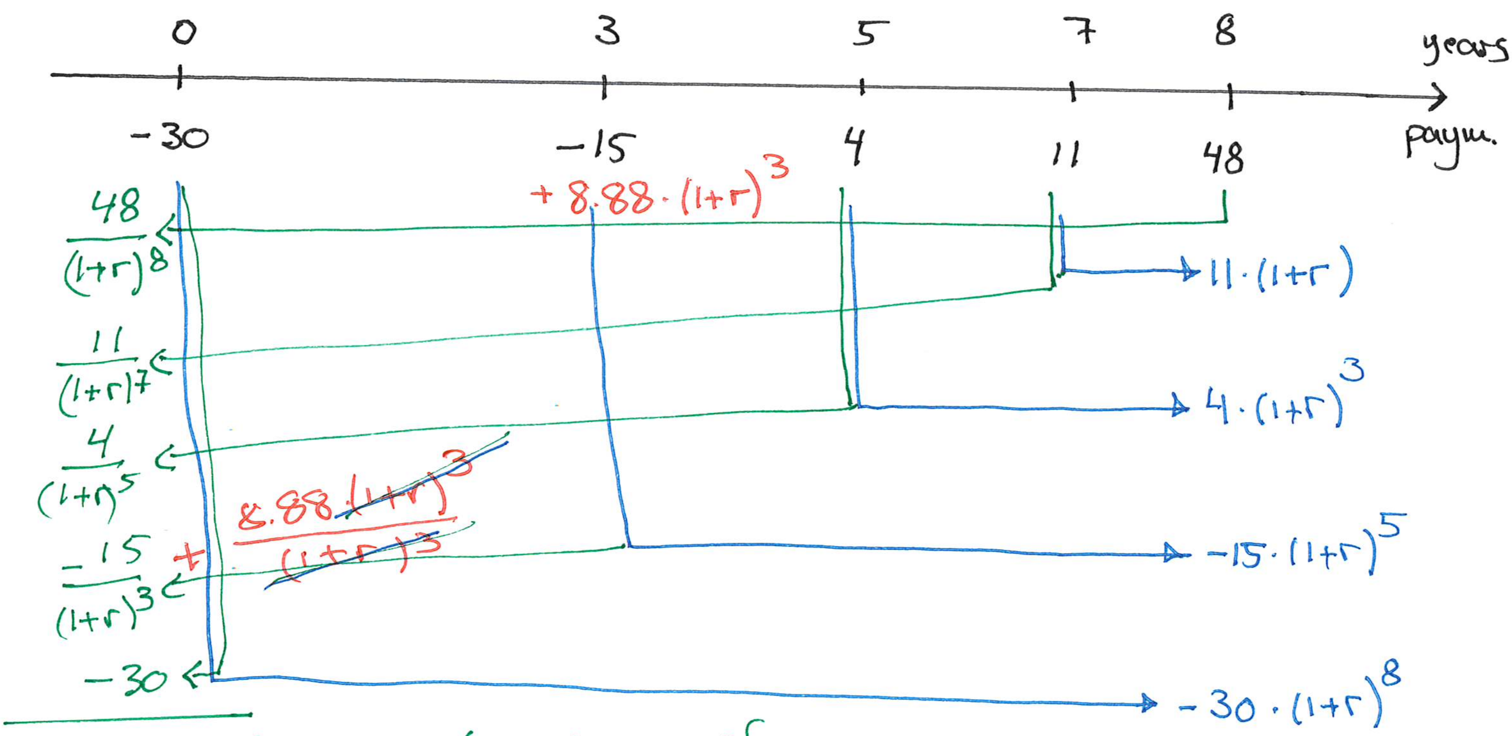
EBA1180, lecture 4, 6 Sept. 2022, Runar Ile

Plan
1. Rep: Total present value of a cash flow
2. Geometric series
3. Annuities

1. Rep: Total present value of a cash flow

Prob. 8 Let r be the interest. The cash flow:



Sum = tot. present value of cash flow with interest r

Sum = future value of cash flow 8 yrs. from now with interest r

b) If $r=9\%$ the present value $= -8.88$

d) If $r=13\%$ the pres. value $= -15.49$

a) With $r=9\%$, the future value $= -17.69$

c) With $r=13\%$, the future value $= -41.19$

Observation $\quad -8.88 \cdot (1+9\%)^8 = -17.69$
tot. pres. value $\quad -15.49 \cdot (1+13\%)^8 = -41.18$ why?

$$K_0 \cdot (1+r)^8 = \left[-30 - \frac{15}{(1+r)^3} + \frac{4}{(1+r)^5} + \frac{11}{(1+r)^7} + \frac{48}{(1+r)^8}\right] \cdot (1+r)^8$$

$$= -30 \cdot (1+r)^8 - 15 \cdot (1+r)^5 + 4 \cdot (1+r)^3 + 11 \cdot (1+r) + 48$$

$$= K_8$$ – the future value 8 yrs from now with interest r

Problem How much should the payment today (−30) be changed so that the IRR of the (new) cash flow becomes the

i) 9 % ? New payment today: $-30 + 8.88 = \underline{\underline{-21.12}}$

ii) 13 % ? —— " —— $-30 + 15.49 = \underline{\underline{-14.51}}$

How should the payment 8 yrs. from now (48) be changed so that the future value of the (new) cash flow becomes 0 if

iii) $r = 9\%$? New payment 8 yrs from now: $48 + 17.69 = \underline{\underline{65.69}}$

iv) $r = 13\%$? —— " ——: $48 + 41.19 = \underline{\underline{89.19}}$

Start: 11.04

2. Geometric series

A series : - many terms are added

third term

Ex $1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{100}$ is a series

with 10 terms

We write $a_1 + a_2 + a_3 + \dots + a_{10}$

Geometric series $a_1 + a_2 + \dots + a_n$

where each term is k times the previous term (k is a number)

$$a_2 = k \cdot a_1$$

$$a_3 = k \cdot a_2 = k \cdot (k \cdot a_1) = k^2 \cdot a_1$$

$$a_4 = k \cdot a_3 = k \cdot (k^2 \cdot a_1) = k^3 \cdot a_1$$

$$\vdots$$

$$a_{10} = k^9 \cdot a_1$$

We can find a short expression for the series (the sum) :

$$a_1 + a_2 + \dots + a_n = a_1 + a_1 k + a_1 k^2 + \dots + a_1 \cdot k^{n-1}$$

$$= a_1 (\underbrace{1 + k + k^2 + \dots + k^{n-1}}_{\frac{k^n - 1}{k - 1}})$$

$$= a_1 \cdot \frac{k^n - 1}{k - 1}$$

Problem Compute the sum

$$5 + 5\cdot 1.003 + 5\cdot 1.003^2 + 5\cdot 1.003^3 + \dots + 5\cdot 1.003^{60}$$

Solution This is a geometric series with $a_1 = 5$ and $k = 1.003$ and the number of terms $n = 61$ so the sum is

$$5 \cdot \frac{1.003^{61} - 1}{1.003 - 1} = 5 \cdot \frac{1.003^{61} - 1}{0.003} = \underline{\underline{334.14}}$$

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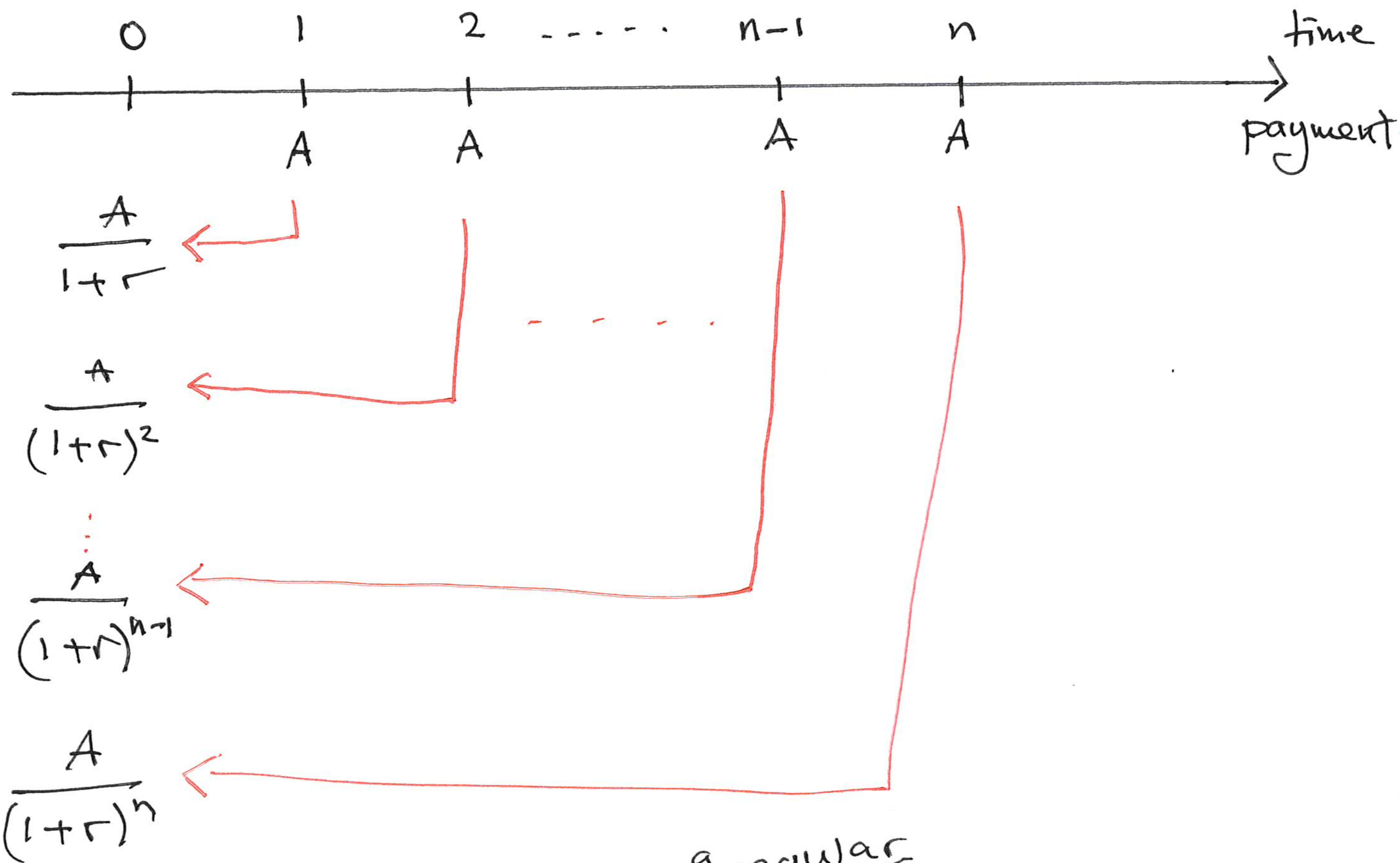
3. Annuities – regular cash flows



Sum = tot. pres. val. of a regular cash flow. It is a geometric series with $a_1 = \frac{A}{1+r}$ and $k = \frac{1}{1+r}$.

Then the sum (tot. pres. val.) is

$$\frac{A}{1+r} \cdot \frac{\left(\frac{1}{1+r}\right)^n - 1}{\frac{1}{1+r} - 1}$$

not so nice !

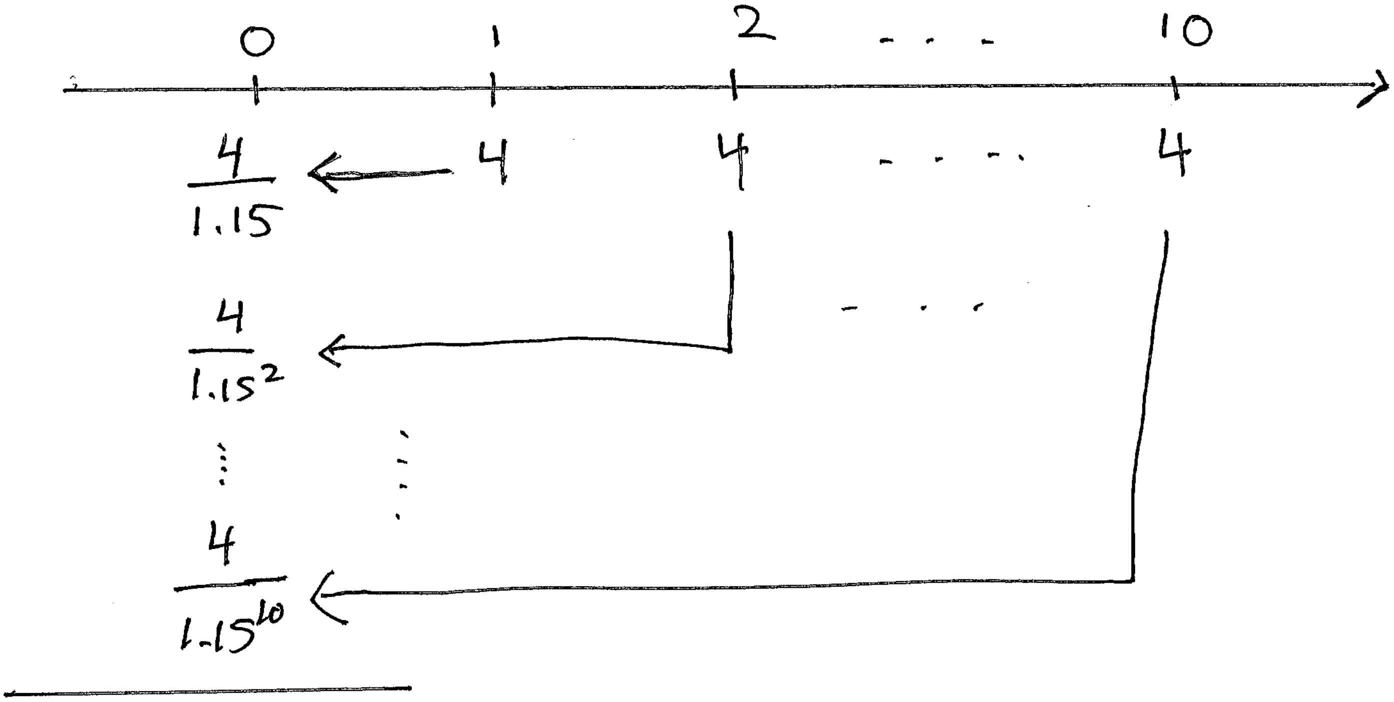
A finite geometric series is also a geom. series in the opposite direction!

Then $a_1 = \frac{A}{(1+r)^n}$, $k = 1+r$ so the sum is also

$$\frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{1+r-1} = \frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{r}$$

Ex Hege considers an investment where 4 mill. is paid out every year for 10 years.
The first payment is one year from now.
Suppose the discount rate is 15 %
What is a fair price for this cash flow ?

We determine the tot. pres. val. of the cash flow.



The sum is a geom. series with

$a_1 = \frac{4}{1.15^{10}}$, $k = 1.15$ and $n = 10$

so the sum (tot. pres. val.) is

$$\frac{4}{1.15^{10}} \cdot \frac{1.15^{10} - 1}{0.15} = \underline{\underline{20.08}}$$