EBA1180, lecture 2, 30 Aug. 2022, Runar Ile

Plan
1. Repetition (alg. exp., roots & powers, absolute value)
2. Relative change and rate of change
3. Interest
4. Present value

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1. Repetition

Fractions $\frac{2}{3} \cdot \frac{5}{4} = \frac{2 \cdot 5}{3 \cdot 4} = \frac{10}{12}$

and $\frac{x+3}{x+4} \cdot \frac{x-1}{x+2} = \frac{(x+3)\cdot(x-1)}{(x+4)\cdot(x+2)}$

Probl. 1i $\frac{18}{4} \cdot \frac{\frac{2}{3}}{12} = \frac{18 \cdot \frac{2}{3}}{4 \cdot 12} = \frac{\frac{18}{1} \cdot \frac{2}{3}}{4 \cdot 12}$

$= \frac{\frac{18 \cdot 2}{1 \cdot 3}}{4 \cdot 12} = \frac{\frac{18 \cdot 2}{3} \cdot 3}{4 \cdot 12 \cdot 3} = \frac{18 \cdot 2}{4 \cdot 12 \cdot 3} = \frac{9 \cdot 2 \cdot 2}{4 \cdot 12 \cdot 3}$

$= \frac{1}{4}$

Probl 2i $\frac{x^2 - 3x}{x(y-3)} \cdot \frac{xy^2 - 9x}{x-3} = \frac{x(x-3)}{x(y-3)} \cdot \frac{x(y^2-9)}{x-3}$

$= \frac{x(x-3)}{x(y-3)} \cdot \frac{x(y-3)(y+3)}{x-3} = \frac{(x-3)\cdot x \cdot (y-3)\cdot(y+3)}{(y-3)\cdot(x-3)}$

$= \frac{x(y+3)}{1} = x(y+3)$

Order of operations

$2 + 3 \cdot 4 = 14$ $\quad -3^2 = (-1) \cdot 3 \cdot 3 = -9$

$(2+3) \cdot 4 = 20$ $\quad (-3)^2 = (-3) \cdot (-3) = 9$

but $-3 \cdot 4 = -12$

Alg/Chain

Roots/Powers

$\sqrt{5} = 5^{0,5} = 5^{\frac{1}{2}}$

$\sqrt{5} \cdot \sqrt{5} = 5^{0,5} \cdot 5^{0,5} = 5^{0,5+0,5} = 5^1 = 5$

$\sqrt[3]{5} = 5^{\frac{1}{3}}$

so $\left(\sqrt[3]{5}\right)^6 = \left(5^{\frac{1}{3}}\right)^6 = 5^{\frac{1}{3} \cdot 6} = 5^2$

Moreover: $5^{-1} = \frac{1}{5}$

and $5^{-2} = \frac{1}{5^2}$

Pattern If m, n integers, $n > 0$ and $a > 0$ (pos. number), then

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Probl 6i $\quad \frac{\sqrt{1.03}^{10}}{1.03^4} = \frac{\left(1.03^{\frac{1}{2}}\right)^{10}}{1.03^4} = \frac{1.03^5}{1.03^4}$

$= 1.03^{5-4} = \underline{1.03}$

Problem Calculate $1.11^{\sqrt{2}}$ on your calculator

(answer : 1.159035.....)

Answer 1.11 [y^x] 2 [$\sqrt{x}$] [=]

Same base : $2^{1.5} \cdot 2^{3.8} = 2^{1.5+3.8} = 2^{5.3}$

Same exponent:

$$2^4 \cdot 3^4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

$$= 2 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 3$$

$$= (2 \cdot 3)^4 = 6^4$$

Ex $\sqrt{2} \cdot \sqrt{3} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = (2 \cdot 3)^{\frac{1}{2}} = \sqrt{2 \cdot 3}$

Pattern $a^r \cdot b^r = (ab)^r$

Problem Calculate 1.12^{-1} on your calc.

Solution1 : 1.12 [y^x] 1 [+/-] [=]

Solution2 : 1.12 [1/x] (reason : $1.12^{-1} = \frac{1}{1.12}$)

Absolute value

Ex $\sqrt{(-3)^2} = \sqrt{(-3)\cdot(-3)} = \sqrt{9} = 3 = -(-3) = |-3|$

so $\sqrt{x^2} = |x|$

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

2. Relative change and rate of change

$$\text{Relative change} = \frac{\text{new value} - \text{old value}}{\text{old value}}$$

Recall $\quad \% = \frac{1}{100} = 0.01$

$\quad 3\% = 3 \cdot \frac{1}{100} = 0.03$

Ex Kåre's hourly wage increased from 163 kr to 181 kr. The relative change was

$$\frac{181\,\text{kr} - 163\,\text{kr}}{163\,\text{kr}} = \frac{18}{163} = 11.0\%$$

Rate of change = 1 + relative change

Ex The rate of change in Kåre's hourly wage is $1 + 0.11 = 1.11$

Problem Last year Kåre earned 54 000 kr with 163 kr/hour. If he works as much this year as last year how much will he earn? (with the new wage)

Solution $54000 \cdot 1.11 = 59\,940$

3. Interest

Ex You deposit 40 000 into an account earning 2.3% annual interest. Interest is added after each year (annual compounding of interest)

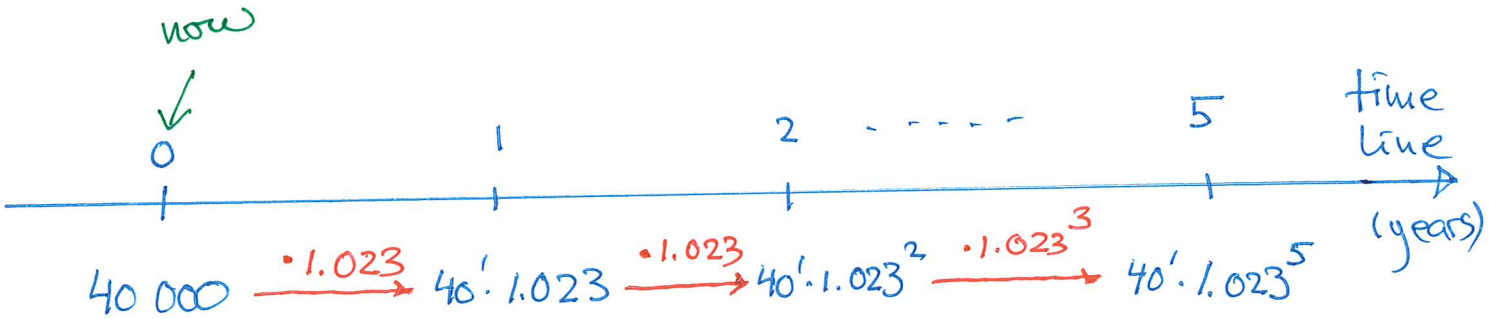
After a year the balance (what's in the account) is

$$40000 + 40000 \cdot 2{,}3\%$$

$$= 40000 \cdot \underbrace{(1 + 2{,}3\%)}_{\text{growth factor}} = \underline{\underline{40920.00}}$$

Problem What is the balance after 5 years?

Solution



$$\underline{40\,000 \cdot 1.023^5} = \underline{44\,816.52}$$

Ex You deposit 40 000 with 2.3% nominal annual interest, but with quaterly compounding of interest.

The growth factor for one period (= 3 months) is

$$1 + \frac{2.3\%}{4} = 1 + 0.575\% = 1.00575$$

0.575% — the interest rate for one period

After 1 year the balance is

$$40\,000 \cdot 1.00575^4 = 40927.96$$

The annual growth factor is

$$1.00575^4 = 1.023199$$

The effective annual interest is

$$1.00575^4 - 1 = 0.023199 = 2{,}3199\,\%$$

Pattern

$$B = B_0 \cdot \left(1 + \frac{r}{n}\right)^m$$

- B: balance after m periods
- B_0: deposit (principal)
- r: nominal interest
- n: number of interest periods in a year
- m: number of periods

4. Present value

Let K_0 be some investment/deposit/payment today. The future value K_n of K_0 in n years (or more generally, n periods) with interest r is

$$K_n = K_0 \cdot (1+r)^n$$

The opposite: Suppose K_n will be paid n years (periods) from now with period interest r. Then the present value K_0 of K_n is given as

$$K_0 = \frac{K_n}{(1+r)^n}$$

Problem 30 mill. is paid 5 years from now with 8% (annual) interest. Determine the present value.

Solution $K_0 = \frac{30 \text{ mill}}{1.08^5} = 20.42 \text{ mill}.$