EBA1180, lecture 19, 8 Nov. 2022, Runar Ile

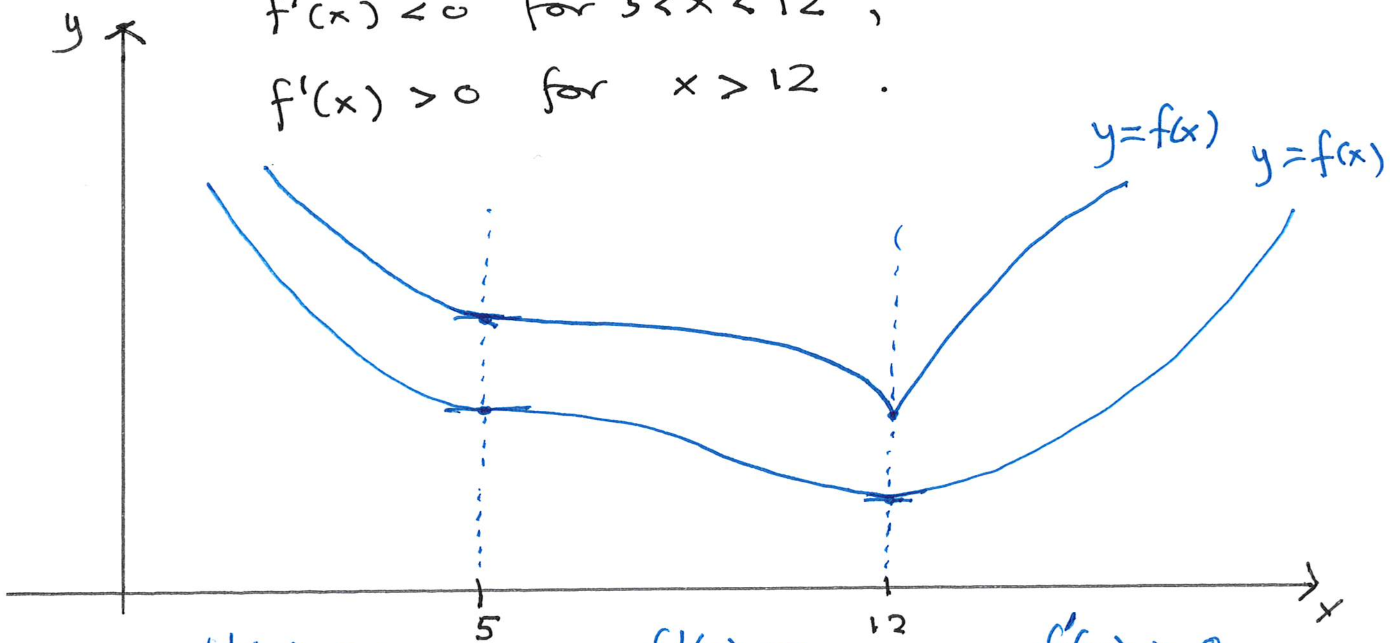
Plan: 1. Repetition with problems from last week:
- Probl. 1c : draw two graphs
- Probl. 2 b, d, h, i, k : interpretations of the graph of $f'(x)$.
- Probl 3c : which graph is $f(x)$ / $f'(x)$?
- Probl 4g : increasing/decreasing from $f'(x)$.

2. Implicit differentiation

Probl. 1c $f'(x) < 0$ for $x < 5$, $f'(5) = 0$

$f'(x) < 0$ for $5 < x < 12$,

$f'(x) > 0$ for $x > 12$.



$f'(x) < 0$ $f'(5) = 0$ $f'(x) < 0$ $f'(x) > 0$

f(x) decreasing f(x) decreasing f(x) increasing

Probl 2 b) $f(2) < f(3)$? FALSE

We see (from the graph of $f'(x)$) that

$f'(x) < 0$ for $x \in [2, 3]$.

Hence $f(x)$ is strictly decreasing for $x \in [2, 3]$

and $f(2) > f(3)$.

Problem 2 I figure 1 you see the graph of $f'(x)$.

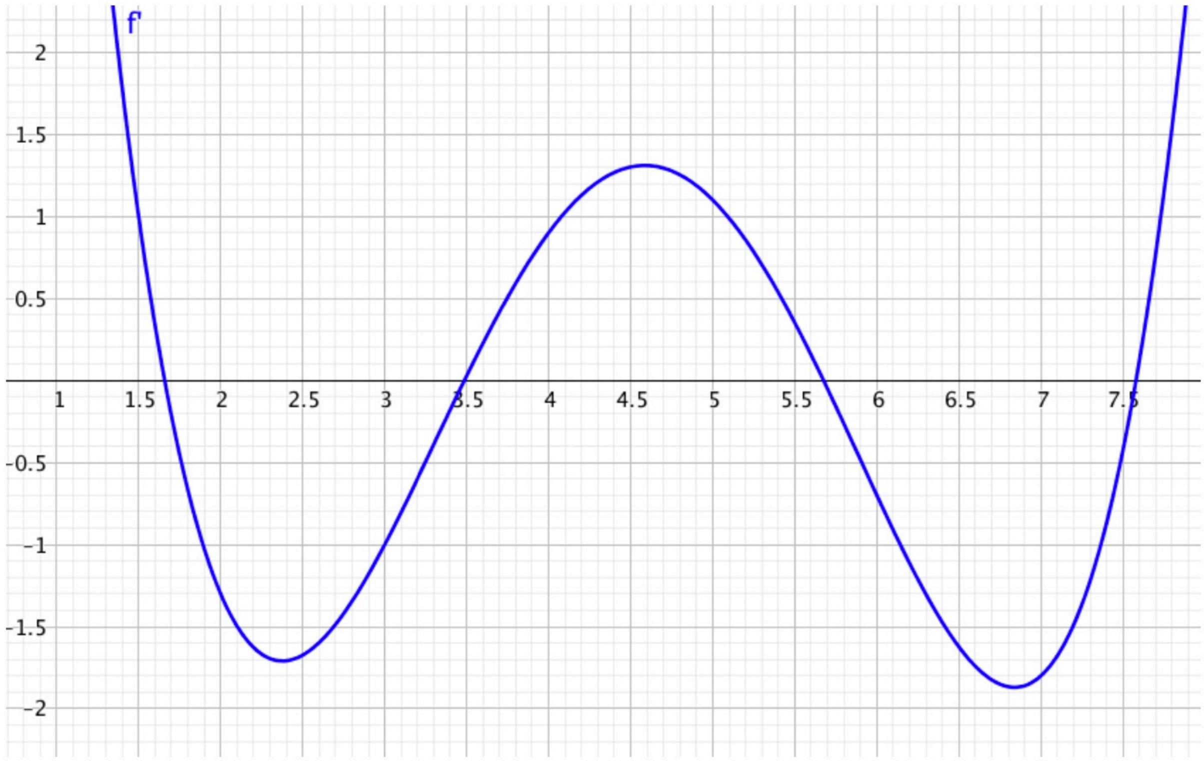


Figure 1: The graph of $f'(x)$

Determine if the statement is true or false.

2d) $f(x)$ has a (local) minimum at $x = 3.5$

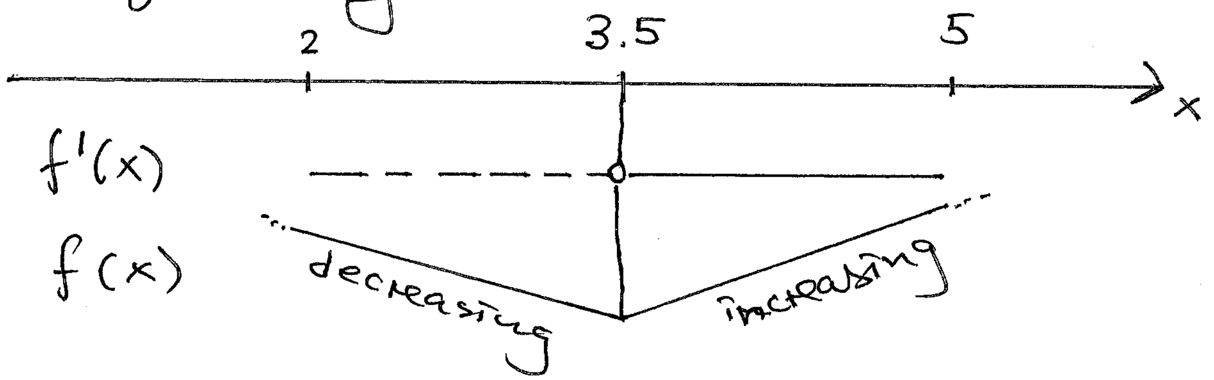
TRUE.

We have $f'(x) < 0$ for $x \in [2, 3.5\rangle$

and $f'(x) > 0$ for $x \in \langle 3.5, 5]$

and $f'(3.5) = 0$.

Sign diagram:



Conclusion: $x = 3.5$ is a loc. min. point for $f(x)$.

2h) $f(x)$ increases faster around $x = 1.5$ than around $x = 5.5$. TRUE.

The slope of the tangent of $f(x)$ at $x = 1.5$ is approx. 1 (since $f'(1.5) \approx 1$).

The slope of the tangent of $f(x)$ at $x = 5.5$ is approx. 0.35 (since $f'(5.5) \approx 0.35$)

2i) The derivative of $f'(x)$ is pos. for $x = 7.6$

TRUE because the slope of the tangent of $f'(x)$ is (very) positive for $x = 7.6$.

(maybe $f''(7.6) \approx 6$).

2k) We cannot use the graph of $f'(x)$ to determine if $f(4.5)$ is positive.

TRUE: If we add 1 mill to $f(x)$ or subtract 1 mill from $f(x)$

$f'(x)$ is not changed.

Probl 3c Which graph is $f(x)$ / $f'(x)$?

I guess $f(x)$ is the violet one. But (much) easier to determine what is wrong!

Assume $f(x)$ is the green.

Then $f'(x)$ is the violet one.

But the slope of the green is negative for $x > 3$ while the violet one is bigger than 1. So the assumption is wrong, and the only possibility is that $f(x)$ is the ~~violet~~ violet one, and $f'(x)$ is the green.

Start: 11.02

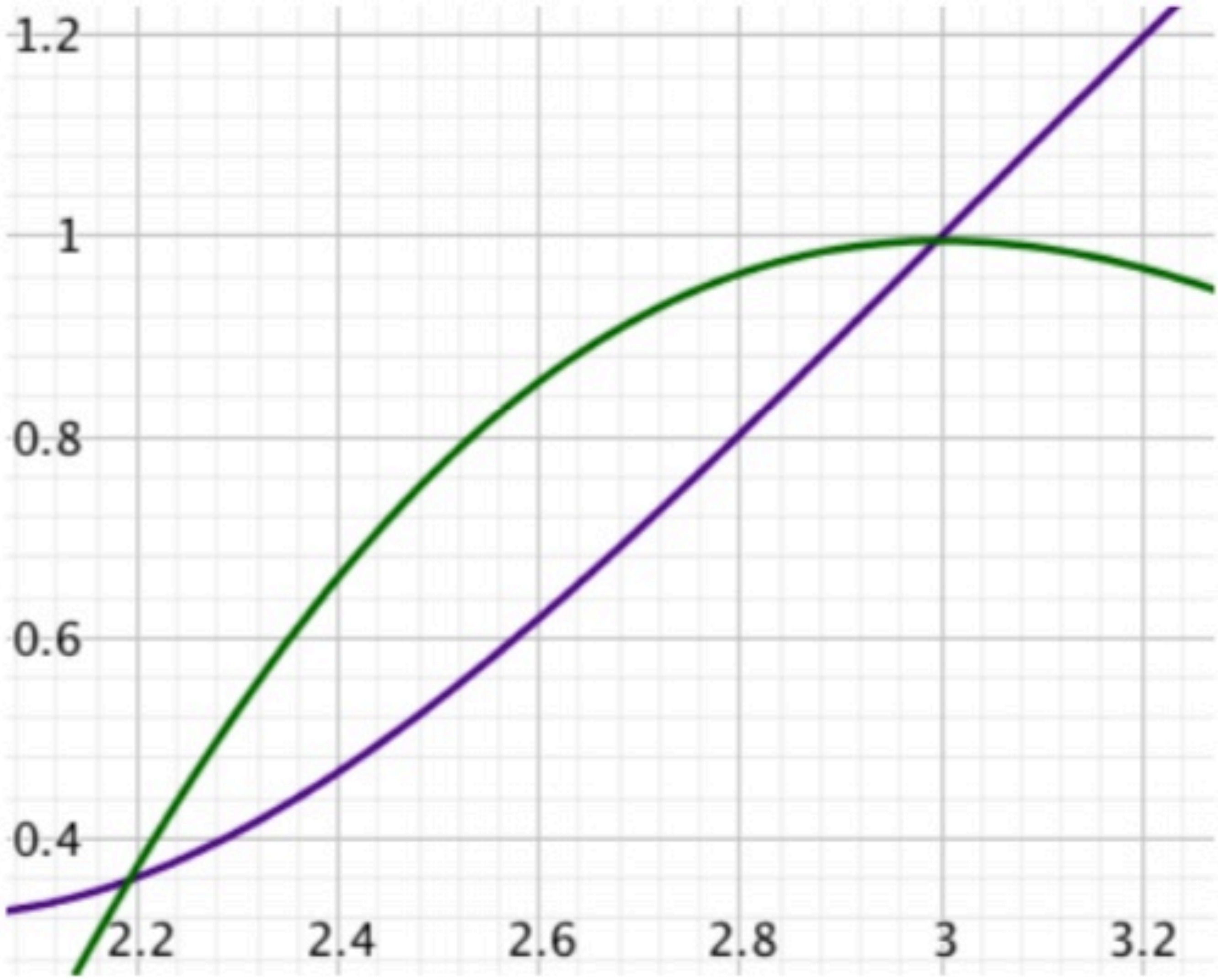
Probl 4g $f'(x) = e^{2x} - 4e^{x} + 3$. When is $f(x)$ strictly increasing / decreasing ?

We want to use the sign diag. of $f'(x)$.

But have to factorise $f'(x)$ first.

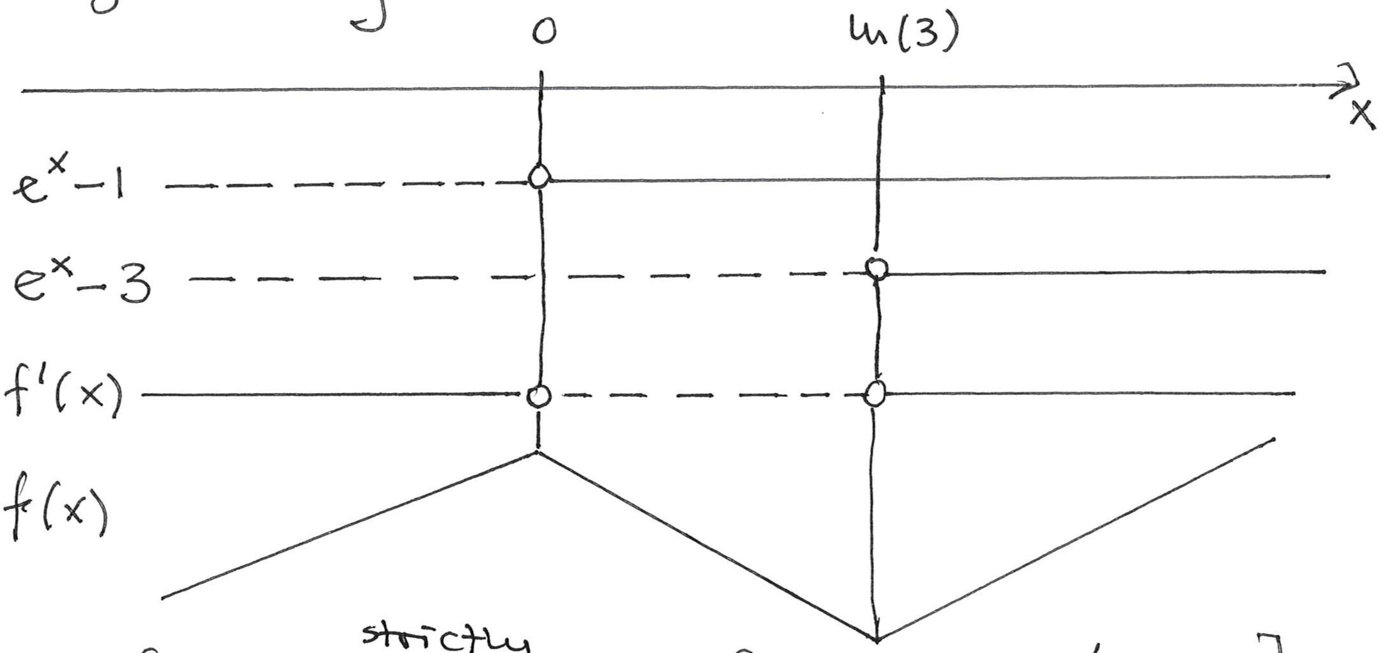
Put $u = e^x$. Then $u^2 = e^x \cdot e^x = e^{2x}$. So

$f'(x) = u^2 - 4u + 3 = (u-1)(u-3)$


1.2
1
0.8
0.6
0.4
2.2
2.4
2.6
2.8
3
3.2

so $f'(x) = (e^x - 1)(e^x - 3)$.

Sign diag.



so $f(x)$ is strictly increasing for x in $\langle \leftarrow, 0]$

$f(x)$ is strictly decreasing for x in $[0, \ln(3)]$

$f(x)$ is strictly increasing for x in $[\ln(3), \rightarrow\rangle$

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2. Implicit differentiation

EX $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = (-1) \cdot x^{-1-1} = -x^{-2} = -\frac{1}{x^2}$

– usual differentiation.

Instead put $y = f(x)$, so $y = \frac{1}{x}$ $\quad \mid \cdot x$

and get $\boxed{xy = 1}$

Differentiate each side of the eq. with respect to x and think about y as a function of x

$$(x\cdot y)'_x = (1)'_x$$

the product rule on the left hand side gives:

$$(x)'_x \cdot y + x\cdot (y)'_x = 0$$

$$1\cdot y + x\cdot y' = 0$$

We can solve this eq. for y':

$$x\cdot y' = -y \quad \Big| : x$$

$$\boxed{y' = -\frac{y}{x}}$$

(Note: $y = \frac{1}{x}$, so $y' = -\frac{\left(\frac{1}{x}\right)}{x} = -\frac{1}{x^2}$)

This is called implicit differentiation.

<u>One application</u> Can use this to find slopes of tangents to curve defined by the original equation (so $xy = 1$)

E.g. If $x = 2$ then $xy = 1$ gives $2y = 1$

so $y = \frac{1}{2}$

Also $y'\Big|_{\substack{x=2\\ y=\frac{1}{2}}} \overset{\text{red box}}{=} -\frac{\frac{1}{2}}{2} = -\frac{1}{4}$

Can apply this to find the function expression $h(x)$ of the tangent at the point $(2, \frac{1}{2})$ by the point-slope formula:

$$h(x) - \frac{1}{2} = -\frac{1}{4}(x - 2)$$

the slope

so $$\underline{\underline{h(x) = -\frac{1}{4}x + 1}}$$