EBA 1180, lecture 15, 25 Oct. 2022, Runar Ile

Plan: Talk about some of the course paper problems

Probl. 9ab Inverse functions
Probl. 10 An increasing function
Probl. 8bc Ellipses
Probl. 6b Polynomial division and factorization
Probl. 7b Hyperbola functions

Probl 9 Inverse functions

9a) $f(x) = 10 + \frac{0.2}{x-3}$, $D_f = \langle 3, \infty \rangle$

To find the inverse function with expression $g(x)$ and domain of definition D_g we:

① Solve the equation $y = f(x)$ for x

$$y = 10 + \frac{0.2}{x-3} \quad \Big| -10$$

$$y - 10 = \frac{0.2}{x-3} \quad \Big| \cdot (x-3)$$

$$(y-10)(x-3) = 0.2 \quad \Big| : (y-10)$$

$$x - 3 = \frac{0.2}{y-10} \quad \Big| +3$$

$$x = 3 + \frac{0.2}{y-10}$$

② Switch variables: $g(x) = 3 + \frac{0.2}{x-10}$

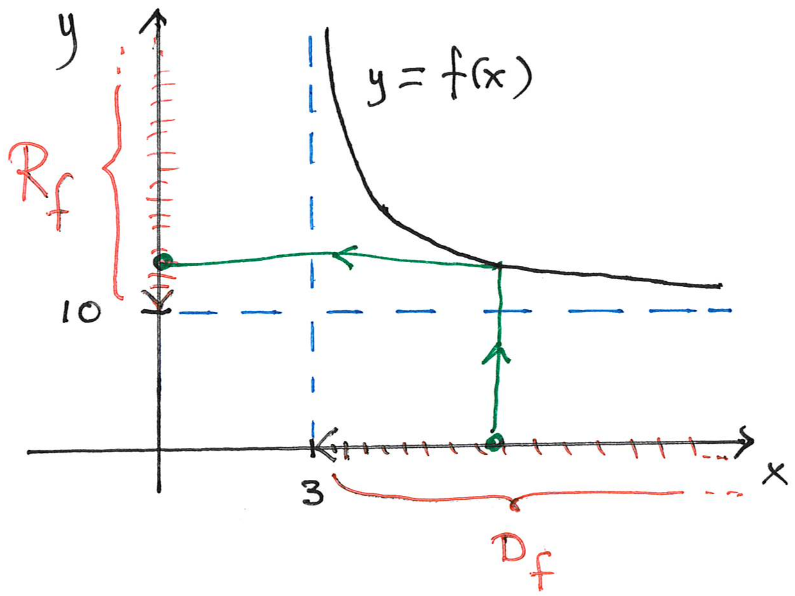
③ Always: $D_g = R_f$ (the range of $f(x)$).

$f(x)$ has a vertical asymptote for $x = 3$

and $f(x) \xrightarrow[x \to 3^+]{} +\infty$

$f(x)$ also has a horizontal asymptote $y = 10$

and $f(x) \xrightarrow[x \to \infty]{} 10^+$ so



$D_g = R_f = \langle 10, \infty \rangle$

9b) $f(x) = \ln(10x - x^2)$, $D_f = [1, 5]$

① Solve $y = \ln(10x - x^2)$ for x

Insert LHS and RHS into $e^{(-)}$

$$e^y = e^{\ln(10x - x^2)} = 10x - x^2$$

$$x^2 - 10x = -e^y$$

Complete the square

$$(x-5)^2 = 25 - e^y \quad | \sqrt{\ }$$

$$|x-5| = \sqrt{25 - e^y}$$

since $1 \le x \le 5$ we have $-4 \le x-5 \le 0$

so $-(x-5) = \sqrt{25-e^y}$ for $x \in D_f$

that is $x = 5 - \sqrt{25 - e^y}$

(2) Switch variables: $\underline{\underline{g(x) = 5 - \sqrt{25 - e^x}}}$

(3) $D_g \overset{\text{always}}{=} R_f$ and $f(1) = \ln(10 \cdot 1 - 1^2) = \ln(9)$

$f(5) = \ln(10 \cdot 5 - 5^2) = \ln(25)$

Since the eq. in (1) has a solution for all y in this interval,

$$D_g = R_f = \underline{\underline{[\ln(9), \ln(25)]}}$$

Probl 10: An increasing function

We show that $f(x) = e^x$ is increasing by using the definition.

Assume $x_1 < x_2 \quad | -x_1$

$0 < x_2 - x_1$

$1 < e^{x_2 - x_1}$

$\boxed{\text{Given fact: } e^x > 1 \text{ for } x > 0.}$

that is $1 < \frac{e^{x_2}}{e^{x_1}}$ $\quad | \cdot e^{x_1} > 0$

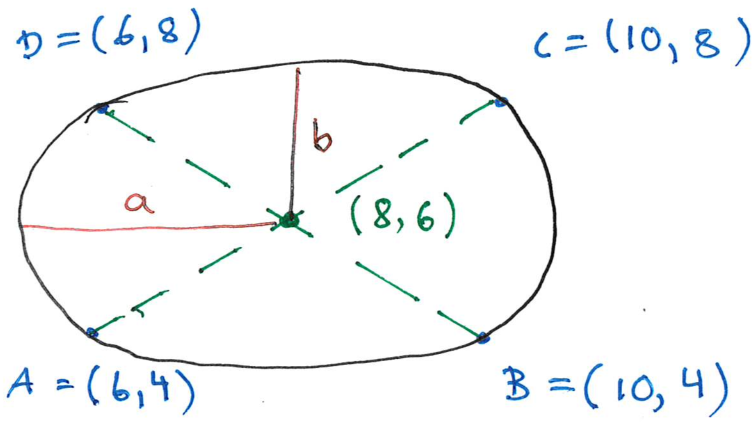
$$f(x_1) = e^{x_1} < e^{x_2} = f(x_2)$$

Hence $f(x)$ is a strictly increasing function for all x.

Start : 11.00

Probl. 8bc Ellipses

8b)



Std. eq. for such an ellipse is

$$\frac{(x-8)^2}{a^2} + \frac{(y-6)^2}{b^2} = 1$$

Note from 8a : If $a = b$, so the ellipse is a circle, then $a = b = \sqrt{8} = 2\sqrt{2} < 3$

So I choose $a = 3$ (I try it!) and since $C = (10, 8)$ is on the ellipse we get the eq. $\frac{(10-8)^2}{9} + \frac{(8-6)^2}{b^2} = 1$ for b.

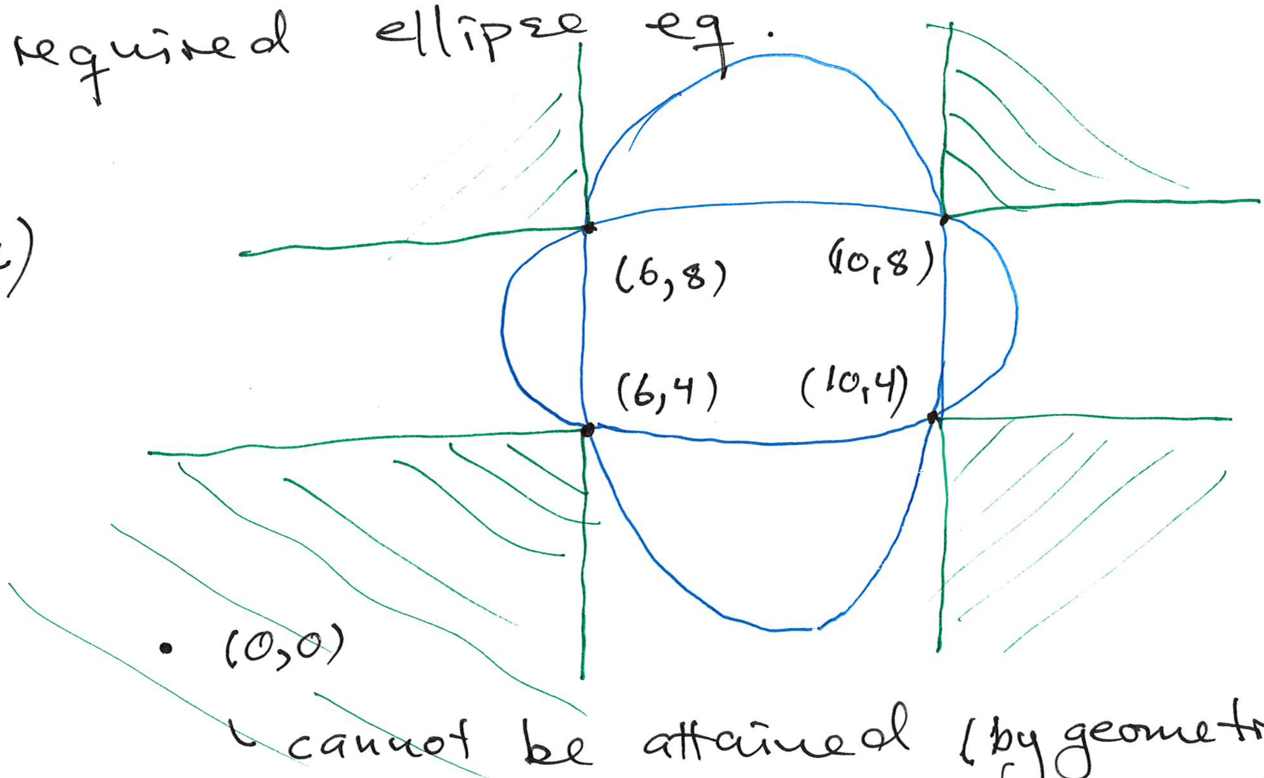
Solve it and get $b^2 = \frac{36}{5} = 7.2$

and $b = \frac{6}{\sqrt{5}} < 3$

So $\underline{\underline{\frac{(x-8)^2}{9} + \frac{(y-6)^2}{7.2} = 1}}$ is a

required ellipse eq.

8c)



cannot be attained (by geometry)

<u>By algebra</u>: If (0, 0) is on the ellipse

$$\frac{(x-8)^2}{a^2} + \frac{(y-6)^2}{b^2} = 1$$ then

$$\frac{(0-8)^2}{a^2} + \frac{(0-6)^2}{b^2} = 1$$, that is

$$\boxed{\frac{64}{a^2} + \frac{36}{b^2} = 1}$$

But C is also on the ellipse, so

$$\frac{(10-8)^2}{a^2} + \frac{(8-6)^2}{b^2} = 1$$

that is

$$\boxed{\frac{4}{a^2} + \frac{4}{b^2} = 1}$$

subtract LHS and RHS:

$$\frac{60}{a^2} + \frac{32}{b^2} = 0$$

Since the LHS always is a pos. number, there are no solutions for a and b, and our assumption is wrong:

<u>Conclusion</u>: No such ellipse passes through the origin (0, 0).

Probl. 6b Polynomial division and factorization

By polynomial division we got

$$\underset{\substack{\text{given} \\ \text{3rd deg. poly.}}}{f(x)} = \underset{\text{quad. poly.}}{q(x)} \cdot (x-5) + \underbrace{25b+5c}_{\text{constant term}}$$

If $x=5$ then $f(5) = q(5)\cdot\overbrace{(5-5)}^{0} + 25b+5c$

$$= 25b+5c$$

If $x-5$ is a factor in $f(x)$, then $f(x)=h(x)\cdot(x-5)$

so $f(5) = h(5)\cdot 0 = 0$ so $25b+5c = 0$

If $25b+5c=0$ then $f(x) = q(x)\cdot(x-5)$

and $(x-5)$ is a factor of $f(x)$.

So $25b+5c=0$ if and only if $x-5$ is a factor in $f(x)$

divide by 5

$$5b+c=0 \quad \text{———— " ————}$$

Prob 7b: Hyperbola functions

$g(x)$ hyperbola function means that we can write $g(x) = c + \dfrac{a}{x-b}$ for numbers a, b, c.

Then $g(x) \xrightarrow[x\to\infty]{} c$ so $y=c$ is the horizontal asymptote which is given as $c = 100$

And $g(x) \xrightarrow[x\to b]{} \pm\infty$ so $x=b$ is the vertical asymptote, given as $b=0$ (the y-axis)

So $g(x) = 100 + \frac{a}{x}$

What is a? - Have $g(5) = 98$, that is

$$100 + \frac{a}{5} = 98$$

so $\underline{a = -10}$ so $\underline{\underline{g(x) = 100 - \frac{10}{x}}}$