
 Plan

- 1 Summary: Quadratic functions
 - 2 Problem set 10: Problem 5,7,8,9
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① Quadratic functions

$$\begin{aligned}
 f(x) &= ax^2 + bx + c \quad (a \neq 0) \\
 &= a \cdot (x - r_1)(x - r_2) \\
 &= a \cdot (x - s)^2 + d
 \end{aligned}$$

Standard form

 r_1, r_2 rootsSymmetry line /
(max/min - pt)② Problems from Problemset 10:

5a) $f(x) = a(x-2)(x-5)$ $\leftarrow (x,y) = (0,5)$

$$5 = a(-1)(-5)$$

$$5 = 5a \rightarrow a = 1 \Rightarrow f(x) = \underline{\underline{\frac{1}{2}(x-2)(x-5)}}$$

b) $f(x) = a(x-r_1)(x-r_2)$

$$= a(x-2)(x+3)$$

$$6 = a(-2)(3)$$

$$-6a = 6 \quad a = -1$$

$$r_2 = 2$$

$$r_1 = -3$$

Symmetry line: $x = -1/2$

$$(x,y) = (0,6)$$

$$f(x) = \underline{\underline{-1(x-2)(x+3)}}$$

c) $f(x) = a(x-s)^2 + d$

$$= a(x-100)^2 \quad (x,y) = (90,10)$$

$$10 = a(90-100)^2 = a \cdot 100$$

$$a = \frac{10}{100} = \frac{1}{10}$$

$$\begin{aligned}
 s &= 100 \\
 d &= 0
 \end{aligned}$$

$$f(x) = \underline{\underline{\frac{1}{10}(x-100)^2}}$$

d) $f(x) = a(x-s)^2 + d$ $s=1$ $d=-1$
 $= a(x-1)^2 - 1$
 $-2 = a(-1)^2 - 1$
 $a = -1$ $\rightarrow f(x) = \underline{\underline{-(x-1)^2 - 1}}$

e) $f(x) = a(x-s)^2 + d$ $s=-3$ $d=4.25$
 $= a(x+3)^2 + 4.25$ $\leftarrow (-2, 4.5)$
 $4.5 = a \cdot 1^2 + 4.25$
 $0.25 = a$ $\rightarrow f(x) = 0.25 \cdot (x+3)^2 + 4.25$
 $= \underline{\underline{\frac{1}{4}(x+3)^2 + \frac{17}{4}}}$

f) $f(x) = a(x-s)^2 + d$ $s=50$ $d=1$
 $= a(x-50)^2 + 1$ $\leftarrow (40, 2)$
 $2 = a(40-50)^2 + 1$
 $1 = a \cdot 100$ $a = \frac{1}{100}$ $\rightarrow f(x) = \underline{\underline{\frac{1}{100}(x-50)^2 + 1}}$

7. a) $f(x) = ax^2 + bx + c = ax^2 + bx + 7 = \underline{\underline{x^2 - 4x + 7}}$

P = (0, 7): $7 = a \cdot 0^2 + b \cdot 0 + c$ $c=7$
 Q = (1, 4): $4 = a \cdot 1^2 + b \cdot 1 + 7$ $a+b = -3 \Rightarrow b = -3-a$
 R = (2, 3): $3 = a \cdot 2^2 + b \cdot 2 + 7$ $4a+2b = -4$

$4a + 2(-3-a) = -4$
 $4a - 6 - 2a = -4$
 $2a = 2$

b) $f(x) = a(x-s)^2 + d$ $s=-1$
 $= a(x+1)^2 + d$ (midpoint of $x=-5$ and $x=3$)
 $a=1$ $b=-4$

P = (-5, 65): $65 = a(-4)^2 + d$ $16a + d = 65$
 R = (7, 17): $17 = a(8)^2 + d$ $64a + d = 17$
 $\underline{\underline{-48a = 48}}$ $\rightarrow a = -1$
 $-16 + d = 65$
 $d = 81$

$\Rightarrow f(x) = \underline{\underline{-(x+1)^2 + 81}}$

$$\underline{7} \text{ c) } f(x) = a(x-c)^2 + d$$

$$(+4, -6): -6 = a \cdot (4 - 13/2)^2 - \frac{49}{4}$$

$$-6 = a \cdot (-5/2)^2 - \frac{49}{4} \quad | \cdot 4$$

$$-24 = a \cdot \frac{25}{4} \cdot 4 - \frac{49}{4} \cdot 4$$

$$-24 = 25a - 49$$

$$\frac{25a}{25} = \frac{49 - 24}{25} = \frac{25}{25}$$

$$a = 1$$

goes thr. $(4, -6)$

has min pt. $(13/2, -49/4)$

$$\Downarrow$$

$$(s, d) = (13/2, -49/4)$$

$$a > 0$$

$$f(x) = a(x - 13/2)^2 - 49/4$$

$$\Rightarrow f(x) = \underline{\underline{(x - 13/2)^2 - 49/4}}$$

Want:

$$f(x) = a \cdot (x-s)^2 + d$$

" 3

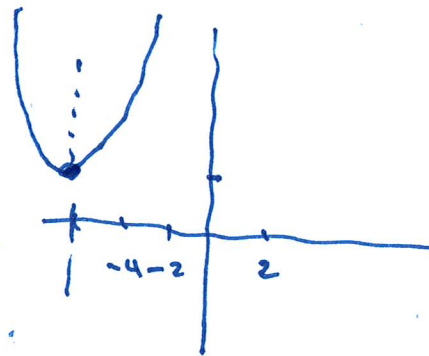
8. b) $f(x) = 3x^2 + 36x + 110$

$$= 3 \left(x^2 + 12x + \left(\frac{12}{2}\right)^2 \right) + 110 - 3 \cdot 36$$

$$\underbrace{x^2 + 12x + 36}_{(x+6)^2}$$

$$= \underline{\underline{3(x+6)^2 + 2}}$$

$$a=3 \quad s=-6 \quad d=2$$



c) $f(x) = -\frac{1}{7}x^2 + 2x - 6$

$$= -\frac{1}{7} \left(x^2 - 14x + 7^2 \right) - 6 + 7$$

$$\underbrace{\hspace{10em}}_{(x-7)^2}$$

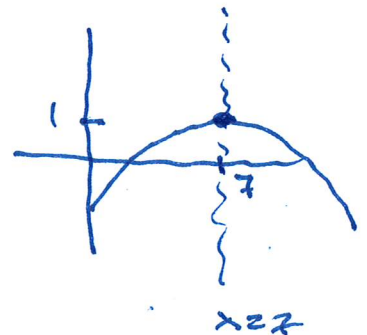
$$= \underline{\underline{-\frac{1}{7}(x-7)^2 + 1}}$$

$$= a(x-s)^2 + d$$

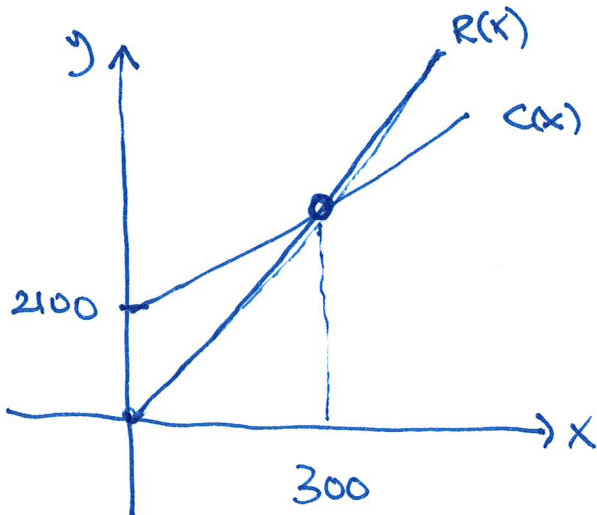
$$a = -\frac{1}{7}$$

$$s = 7$$

$$d = 1$$



$$\begin{aligned} \text{a. a) } R(x) &= px \\ C(x) &= 2100 + 5x \end{aligned} \quad \left. \vphantom{\begin{aligned} R(x) &= px \\ C(x) &= 2100 + 5x \end{aligned}} \right\} \begin{aligned} p(x) &= px - (2100 + 5x) \\ &= (p-5)x - 2100 \end{aligned}$$



$$x < 300: p(x) < 0$$

$$x > 300: p(x) > 0$$

⇓ necessary condition

$$\underline{x = 300: p(x) = 0}$$

$$p(300) = 0$$

$$(p-5) \cdot 300 - 2100 = 0$$

$$p-5 = \frac{2100}{300} = 7$$

$$\underline{p = 12}$$

ok since $p(x) < 0$ for $x < 300$
 $p(x) > 0$ " $x > 300$

$$\text{b) } \left. \begin{aligned} R(x) &= px \\ C(x) &= 4500 - 5x + 0.01x^2 \end{aligned} \right\} 0 \leq x \leq 1000$$

$$\left. \begin{aligned} \text{Given: } x < 300: p(x) < 0 \\ x > 300: p(x) > 0 \end{aligned} \right\} \Rightarrow \text{rec. cond.}$$

$$\begin{aligned} p(x) &= px - (4500 - 5x + 0.01x^2) \\ &= -0.01x^2 + (p+5)x - 4500, \\ &0 \leq x \leq 1000 \end{aligned}$$

$$p(x) = 0 \text{ when } x = 300$$

$$p(300) = 0$$

$$\begin{aligned} -0.01 \cdot 300^2 + (p+5)300 - 4500 &= 0 \\ -900 + (p+5)300 &= 4500 \end{aligned}$$

$$\frac{300(p+5)}{300} = \frac{5400}{300}$$

$$p+5 = 18$$

$$\underline{p = 13}$$

$$\underline{p = 13}$$

$$\begin{aligned} p(x) &= -0.01x^2 + 18x - 4500 \\ &= -0.01(x^2 - 1800x + 900^2) - 4500 + 8100 \\ &= -0.01(x-900)^2 + 3600 \end{aligned}$$

