EBA 1180, lecture 1, 24 Aug. 2022, Runar Ile

Plan
1. Intro. to the Course
2. Algebraic expressions
3. Roots
4. Powers
5. Order of operations
6. Absolute value

---

1. Intro to the Course

Autumn
- Finacial math.
- Functions and graphs
- Differentiation and optimization

Spring
- Integration
- Systems of linear equations
- Functions in two variables

$$z = f(x, y)$$

---

2. Algebraic expressions

Variables: $x, y, z, x_1, x_2, x_3, \dots$

$a, b, c, \dots, m, n$

Multiply with a number

$$3 \cdot x \overset{\text{short form}}{=} 3x$$

$$3 \cdot 2 \neq 32$$

$$\sqrt{3} \cdot x = \sqrt{3}\, x$$

$$(-1) \cdot x = -x$$

$$1 \cdot x = x$$

$$0 \cdot x = 0$$

Addition $\quad x + x = 2x$

$x + y$ no simplification

$$x + y + x = 2x + y$$

Multiplication $\quad x \cdot y = xy$

$$x \cdot x = x^2$$

$$xy \cdot x^2 = x \cdot y \cdot x \cdot x = x^3 y$$

Division $\quad \dfrac{x+4y}{z}, \quad \dfrac{2xy + \sqrt{3}}{3x + y^2}$ ← polynomial
← —"—

Rational expressions : fractions of polynomials

Other expressions : $\sqrt{x^2+1}$, $\dfrac{3\sqrt{x}+1}{\sqrt{x}-1}$

We can insert numbers for the variables:

Ex $\quad \dfrac{2y}{x^2+1}$ with $x = 3$, $y = -1$ gives a

number : $\quad \dfrac{2\cdot(-1)}{3^2+1} = \dfrac{-2}{9+1} = \dfrac{-2}{10} = \dfrac{-1}{5} \doteq -0.2$

– but $\dfrac{2y}{x^2+1}$ cannot be simplified.

Problem We have the rational expression

$$\frac{x^2 - x - 6}{x - 3}$$

a) Fill in

x	1	5	-2	2	8	3
$\frac{x^2-x-6}{x-3}$	3	7	0	4	10	"$\frac{0}{0}$" undefined

b) Find the pattern

Add two to the x-value (except $x=3$)

Shorter: $x+2 \quad (x \neq 3)$

Start: 11.02

Quadratic expansion

$$(x+r)^2 = (x+r)(x+r) = x^2 + 2rx + r^2$$

Ex $(x+5)^2 = x^2 + 10x + 25$

Ex $13^2 = (10+3)^2 = 10^2 + 2 \cdot 3 \cdot 10 + 3^2$

$= 100 + 60 + 9 = 169$

Conjungate expansion

$$(x-r)(x+r) = x^2 - r^2$$

Ex $(x-5)(x+5) = x^2 - 25$

Ex $8 \cdot 12 = (10-2)(10+2) = 10^2 - 2^2$

$= 100 - 4 = 96$

3. Roots

Ex The square root of 5 is the positive number a such that $a \cdot a = 5$

(a is in the calculator: $a = 2.2361\ldots$)

We write a as $\sqrt{5}$

Note: Negative numbers don't have square roots.

Ex $\sqrt{0} = 0$

Problem Compute (without calc.)

a) $(\sqrt{2}+3)^2 = (\sqrt{2})^2 + 2 \cdot \sqrt{2} \cdot 3 + 3^2 = \underline{\underline{11 + 6\sqrt{2}}}$

b) $(\sqrt{5}-1)(\sqrt{5}+1) = (\sqrt{5})^2 - 1^2 = 5 - 1 = \underline{\underline{4}}$

Ex There are other roots:

$\sqrt[3]{5}$ is the number a such that $a \cdot a \cdot a = 5$

$\sqrt[5]{32} = 2$ since $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

4. Powers – repeated multiplication

Ex $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$ "three to the power of four"

$4 \cdot 4 \cdot 4 = 4^3$

exponent

$4^3 \neq 4 \cdot 3$

base

Ex $10^2 \cdot 10^3 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$

$= 10^{2+3}$

so $\boxed{a^n \cdot a^m = a^{n+m}}$

Ex $\frac{3^6}{3^4} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{3 \cdot 3}{1} = 3^2$

$= 3^{6-4}$ (so $3^{-4} = \frac{1}{3^4}$)

$1 = \frac{5^3}{5^3} = 5^{3-3} = 5^0$

$\boxed{(a^n)^m = a^{n \cdot m}}$

Ex $(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$

$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= 3^8 = 3^{2 \cdot 4}$

---

5. Order of operations

Problem Compute

a) $2 + 3 \cdot 4 \overset{?}{=} \begin{cases} 5 \cdot 4 = 20 \\ 2 + 12 = 14 \end{cases}$

b) $2 \cdot 2^4 \overset{?}{=} \begin{cases} 2 \cdot 16 = 32 \\ 4^4 = 64 \end{cases}$

Problem $-5^2 = (-1)\cdot 5\cdot 5 = -25$

---

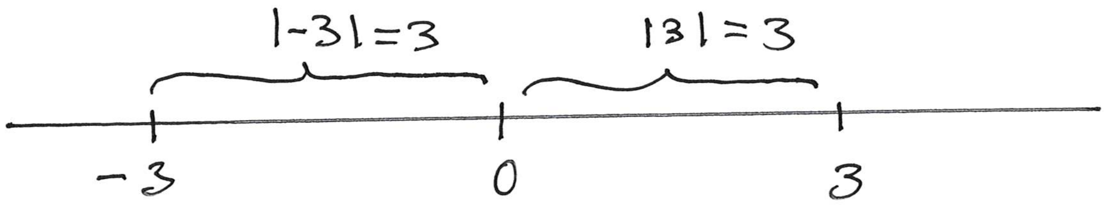
6. Absolute value

If a is a number, then $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

"the absolute value of a"

Ex $|3| = 3$, $|-3| = -(-3) = 3$

$|a|$ = distance between 0 and a on the number line



Problem Simplify $\sqrt{x^2}$

Solution If $x \geq 0$ then $\sqrt{x^2} = x$

If $x < 0$ then $\sqrt{x^2} = -x$

In short: $\sqrt{x^2} = |x|$

Ex: $\sqrt{(x-5)^2} = |x-5|$

Ex: $\sqrt{(-3)^2} = \sqrt{9} = 3$