*... if I couldn't formulate a problem in
economic theory mathematically, I
didn't know what I was doing.*

R. Lucas

Lecture 6
Sec. 4.7, 3.4-5, 2.6
Polynomial division. Factorisation. Rational and radical equations. Inequalities.

Here are recommended exercises from the textbook [SHSC].

Section **4.7** exercise 1-6
Section **3.4** exercise 1, 2
Section **3.5** exercise 2-4
Section **2.6** exercise 1-4, 7

Problems for the exercise session
Wednesday 18 Sept. from 14 o'clock in B2-085

Problem 1 Perform the polynomial division with remainder.

a) $(x^2 + 4x - 21) : (x - 3)$
b) $(x^2 + 4x - 21) : (x - 4)$
c) $(x^3 + x^2 - 23x + 42) : (x + 6)$

d) $(x^3 + x^2 - 23x + 42) : (x + 1)$
e) $(x^4 + x^2 + 1) : (x^2 - x + 1)$
f) $(x^2 + 3x - 7) : (x - a)$

Problem 2 Factor the polynomial in factors of the least possible degree (e.g. by guessing on a zero and perform polynomial division).

a) $x^2 + 4x - 221$
b) $x^3 + 6x^2 - x - 30$
c) $x^3 - 3x^2 + 5x - 15$

d) $x^4 + 10x^3 + 35x^2 + 50x + 24$

Problem 3 Solve the equations.

a) $\dfrac{5x + 1}{x^2 + x + 1} = -2$
b) $\dfrac{x - 1}{x^2 + x + 1} = 1$
c) $\dfrac{1}{1 + \frac{1}{x}} = \dfrac{1}{x}$
d) $\dfrac{(x - 1)(x - 3)}{(x - 2)(x - 4)} = 2$

Problem 4 Determine the values of a such that the equation has solutions.

a) $x^2 + 2ax + 9 = 0$
b) $\dfrac{1}{x + a} = \dfrac{2}{2x + 3}$
c) $\dfrac{(x - 1)(x - 3)}{(x - 2)(x - 4)} = a$

Problem 5 Solve the equations.

a) $\sqrt{2x + 3} = x + 2$
b) $\sqrt{4x + 1} = x - 1$
c) $\sqrt{x + 2} + \sqrt{x - 3} = 5$

d) $\sqrt{2x + 1} - \sqrt{x + 4} = 1$
e) $\dfrac{1}{\sqrt{x} - 1} - \dfrac{1}{\sqrt{x} + 1} = 2$
f) $\dfrac{1}{\sqrt{x} - 1} - \dfrac{1}{\sqrt{x} + 1} = -1$

Problem 6 Determine the values of a such that the equation $\dfrac{1}{\sqrt{x} - 1} - \dfrac{1}{\sqrt{x} + 1} = a$ has solutions.

Problem 7 Solve the inequalities.

1

a) $2x + 3 \leqslant 5x + 2$

b) $-4x + 1 \geqslant x - 1$

c) $x + 2 < 3 + 5x$

d) $(x - 5)(x + 4) < 0$

e) $(2x + 5)(7 - x) \geqslant 0$

f) $\dfrac{(x - 2)(x + 3)}{(x - 5)(x + 4)} < 0$

g) $\dfrac{-5}{(6 - x)(-12 - 3x)} \geqslant 0$

h) $(x - 5)(x + 4) < 10$

i) $(2x + 5)(7 - x) \geqslant 35$

j) $\dfrac{(x - 2)(x + 3)}{(x - 5)(x + 4)} < 1$

k) $\dfrac{-5}{(6 - x)(-12 - 3x)} \geqslant \dfrac{5}{72}$

Problem 8 Determine a such that the inequality has solutions.

a) $x^2 + 6x \leqslant a$

b) $(x + a)^2 < a$

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Problem 1

a) $x + 7$

b) $x + 8 + \dfrac{11}{x - 4}$

c) $x^2 - 5x + 7$

d) $x^2 - 23 + \dfrac{65}{x + 1}$

e) $x^2 + x + 1$

f) $x + a + 3 + \dfrac{a^2 + 3a - 7}{x - a}$

Problem 2

a) $(x - 13)(x + 17)$

b) $(x - 2)(x + 3)(x + 5)$

c) $(x - 3)(x^2 + 5)$

d) $(x + 1)(x + 2)(x + 3)(x + 4)$

Problem 3

a) $x = -3$, $x = -\dfrac{1}{2}$

b) no solutions

c) $x = \dfrac{1}{2} \pm \dfrac{\sqrt{5}}{2}$

d) $x = 4 \pm \sqrt{3}$

Problem 4

a) $a \leqslant -3$ or $a \geqslant 3$

b) $a = \dfrac{3}{2}$

c) All values of a give solutions

Problem 5

a) $x = -1$

b) $x = 6$

c) $x = 7$

d) $x = 12$

e) $x = 2$

f) no solutions

Problem 6

$a \leqslant -2$ or $a > 0$

Problem 7

a) $x \geqslant \frac{1}{3}$, alternative way of writing: $x \in [\frac{1}{3}, \infty\rangle$

b) $x \leqslant \frac{2}{5}$, alternative: $x \in \langle -\infty, \frac{2}{5}]$

c) $x > -\frac{1}{4}$, alternative: $x \in \langle -\frac{1}{4}, \infty\rangle$

d) $-4 < x < 5$, alternative: $x \in \langle -4, 5\rangle$

e) $-\frac{5}{2} \leqslant x \leqslant 7$, alternative: $x \in [-\frac{5}{2}, 7]$

f) $-4 < x < -3$ eller $2 < x < 5$, alternative: $x \in \langle -4, -3\rangle \cup \langle 2, 5\rangle$

g) $-4 < x < 6$, alternative: $x \in \langle -4, 6\rangle$

h) $-5 < x < 6$, alternative: $x \in \langle -5, 6\rangle$

i) $0 \leqslant x \leqslant \frac{9}{2}$, alternative: $x \in [0, \frac{9}{2}]$

j) $x < -7$ eller $-4 < x < 5$, alternative: $x \in \langle -\infty, -7\rangle \cup \langle -4, 5\rangle$

k) $-4 < x \leqslant 0$ eller $2 \leqslant x < 6$, alternative: $x \in \langle -4, 0] \cup [2, 6\rangle$

Problem 8

a) $a \geqslant -9$

b) $0 < a$