

Problem Sheet 5  
DRE 7007 Mathematics

**Problems**

1. Find all the critical points of the function  $f(x,y) = x^4 + 2x^2y^2 + y^4 - x^2 - y^2$  defined on  $D = \mathbb{R}^2$ , and classify the critical points as local maxima, local minima and saddle points. Does  $f$  have a global maximum or minimum on  $D$ ?

2. Consider the function  $f : D \rightarrow \mathbb{R}$ , defined by  $f(x,y) = \ln(1 - x^2 - y^2)$  on the open set  $D = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ . Is  $f$  concave? Is it quasi-concave? Find  $\max f(x,y)$  when  $(x,y) \in D$ .

3. Consider the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ , defined by  $f(x,y) = 3x^4 + 3x^2y - y^3$ . Find all the critical points of  $f$  and classify their type. Is there a global maximum or a global minimum for  $f$ ?

**Keep answers as short and to the point as possible. Answers must be justified.**