BI NORWEGIAN BUSINESS SCHOOL

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Eksamen 05/2016 MET11802

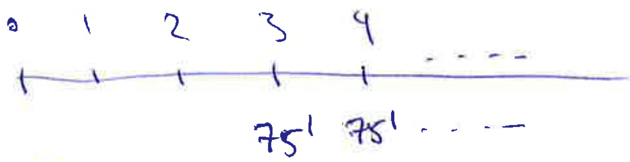
1. Pris etter 10 år:

$$2.452.500 \cdot 1{,}0725^{10} \simeq \underline{4.938.351{,}80} \text{ kr}$$

Dobbel kjøpspris: $2 \cdot 2.452{,}500 = \underline{4.905.00}$ kr

Riktig svar: D Mer enn det dobbelte av kjøpsprisen.

2. Kontantstrøm



Nåverdi: $\frac{75.000}{(1+r)^3}$

$+ \frac{75.000}{(1+r)^4}$

$+ \ldots$

$$= a_1 \cdot \frac{1}{1-k} = \frac{75.000}{(1+r)^3} \cdot \frac{1}{1-\frac{1}{1+r}} = \frac{75.000}{(1+r)^3-(1+r)^2}$$

$r = 0{,}08$

$$= \frac{75000}{(1+r)^2 \cdot r} \simeq \underline{803.755{,}14}$$

Riktig svar: B Mellom 800.000 kr og 850.000 kr.

3. $S = 4 - x + x^2/4 - \ldots$ geometrisk, $k = \frac{-x}{4} = -\frac{x}{4}$ $\frac{a_2}{a_1}$

Konvergent: $-1 < k < 1$

$-1 < -\frac{x}{4} < 1 \quad | \cdot (-4)$.

$4 > x > -4$

Dvs $(-4, 4)$ konvergent for $x = 3$

$S(3) = a_1 \cdot \frac{1}{1-k}$

$= 4 \cdot \frac{1}{1+3/4}$

$= \frac{4}{7/4} = \underline{\frac{16}{7}}$

Riktig svar: B Konvergent for $x=3$ med $S(3) > 0$

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4. Nåverdi :

Merk : kontinuerlig disk. rente 6% $\rightsquigarrow \cdot e^{-10\cdot 0{,}06} = e^{-0{,}6}$

$$160 \cdot e^{\sqrt{10}/3} \cdot e^{-0{,}6} = 459{,}10 \cdot 0{,}5488 \approx \underline{251{,}96} \text{ mill kr}$$

$$= \frac{160\, e^{\sqrt{10}/3}}{e^{0{,}6}}$$

Riktig svar: (A) Mindre enn 300 mill. kr.

5. $x^3 - 4x = x^2 - 4$

Alt 1: Ser at x^2-4 er faktor på begge sider

$x \cdot (x^2-4) = x^2-4$

$x(x^2-4) - (x^2-4) = 0$

$(x-1)(x^2-4) = 0$

$\underline{x=1}$ eller $x^2-4=0$

$\underline{x=2}$, $\underline{x=-2}$

Alt2: $x^3 - x^2 - 4x + 4 = 0$

Mulige heltallsløsn.
$x = \pm 1, \pm 2, \pm 4$

Ser at $x=1$ er løsn.

polynom divisjon $\rightarrow$ $x^3 - x^2 - 4x + 4 = (x-1)\cdot(x^2-4)$

$(x-1)(x^2-4) = 0$

$\underline{x=1}$ eller $x^2 = 4$

$x = \underline{2}, \underline{-2}$

$$(x^3 - x^2 - 4x - 4) : (x-1) = x^2 - 4$$

$\underline{x^3 - x^2}$

$-4x-4$

$\underline{-4x-4}$

0

Riktig svar: (B) $x = 1, 2$ positive (to), $x = -2$ negativ (en)

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6.

$4-\sqrt{x} = x+2 \quad | -4$

$-\sqrt{x} = x-2 \quad | (\cdot)^2$

$x = (x-2)^2$

$x = x^2-4x+4$

$0 = x^2-5x+4$

$x = \frac{5\pm\sqrt{25-16}}{2}$

$= \frac{5\pm 3}{2} = 4, 1$

Prøve: $x=4$: VS $=2$, HS $=6$

$x=1$: VS $=3$, HS $=3$

Løsn: $x=1$

Riktig svar: D   Kun én pos. løsn $x=1$

7.

$\frac{x^2}{3-2x-x^2} > 1$

$\frac{x^2}{3-2x-x^2} - 1 > 0$

$\frac{x^2-(3-2x-x^2)}{3-2x-x^2} > 0$

$\frac{2x^2+2x-3}{3-2x-x^2} > 0$

Fakt: $2x^2+2x-3=0$

$x = \frac{-2\pm\sqrt{4+24}}{4}$

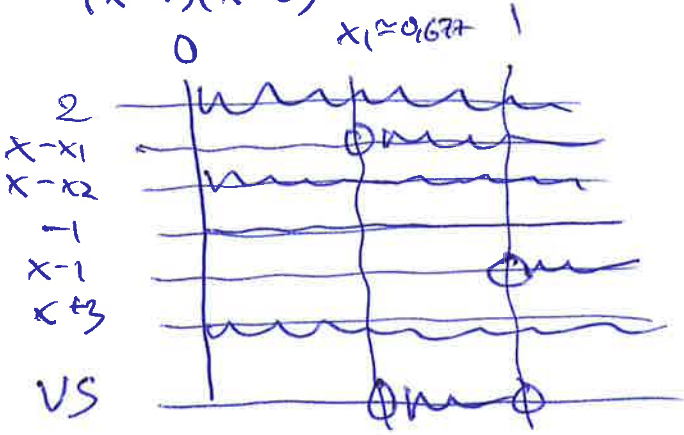
$= -2 \pm \frac{1}{2}\sqrt{7} = x_1, x_2$

$3-2x-x^2=0$

$x = \frac{2\pm\sqrt{4+12}}{-2} = -1\pm 2$

$= 1, -3$

$\frac{2(x-x_1)(x-x_2)}{-(x-1)(x+3)} > 0$ ser på $x>0$ $x_1 \simeq 0{,}677$, $x_2 < 0$



Løsn: VS >0

x i $(x_1, 1)$

$= \left(-2+\frac{1}{2}\sqrt{7}, 1\right)$

Riktig svar: C

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8. $f(x) = x^3 - 4ax - 3$

$f(3)=0$: $3^3 - 4 \cdot a \cdot 3 - 3 = 0$

$27 - 12a - 3 = 0$

$24 - 12a = 0$

$a = 2 \longrightarrow f(x) = x^3 - 8x - 3$

Vet at $x=3$ er nullpkt, bruker polynomdiv. $\longrightarrow$ $= (x-3)\cdot(x^2+3x+1)$

$$x^3 - 8x - 3 : x - 3 = x^2 + 3x + 1$$
$$-(x^3 - 3x^2)$$
$$3x^2 - 8x - 3$$
$$-(3x^2 - 9x)$$
$$x - x$$

Andre nullpkt:

$x^2 + 3x + 1 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4 \cdot 1}}{2} = \frac{-3 \pm \sqrt{5}}{2}$

$x = x_1 = \frac{-3+\sqrt{5}}{2} < 0$

$x = x_2 = \frac{-3-\sqrt{5}}{2} < 0$

Riktig svar: (c) to neg. nullpkt til

9. Vertikale:

$x^2 - 4x + b = 0$; $x = 2$

$\Downarrow$

$2^2 - 4 \cdot 2 + b = 0$

$4 - 8 + b = 0$

$b = 4$

Horisontale/skrå:

$$x^3 - ax^2 + x - 1 : x^2 - 4x + b = x + 4 - a$$
$$-(x^3 - 4x^2 + bx)$$
$$(4-a)x^2 + (1-b)x - 1$$
$$\vdots$$

Skrå $y = x + (4-a) = x + 1$

$4 - a = 1$

$a = 3$

$ab = 3 \cdot 4 = 12$

Riktig svar: (A) $a>0, b>0$

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10. $f = x^2 e^{2-x} - e\ln(\sqrt{e})$ ← konstant (uten x)

$f' = 2x e^{2-x} + x^2 \cdot e^{2-x} \cdot (-1) - 0$

$f'(2) = 4e^0 + 4e^0(-1) = \underline{0}$

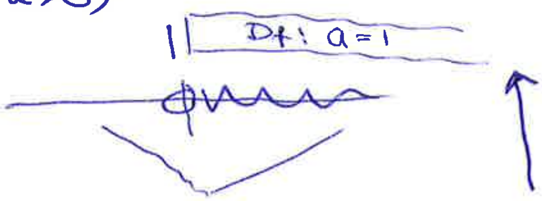
Riktig svar: C $\quad f'(2) = 0$

11. $f = x\ln x - x + 3$, $x \geq a$ $(a > 0)$

$f' = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1$

$= \ln x + 1 - 1$

$= \ln x$

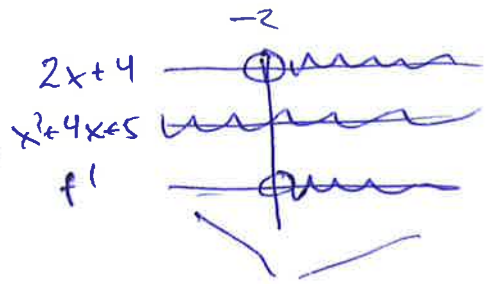


$a = 1 \Rightarrow D_f = [1, \to) \Rightarrow f$ voksende
f^{-1} fins

Riktig svar: B $\quad a = 1$

12. $f = \ln(x^2 + 4x + 5)$

$f' = \frac{1}{x^2+4x+5} \cdot (2x+4) = \frac{2x+4}{x^2+4x+5}$



Faktor: $x^2 + 4x + 5 = 0$

$x = \frac{-4 \pm \sqrt{16-20}}{2}$

Ingen løsn $\Rightarrow x^2 + 4x + 5$ alltid pos.
siden $x = 0$ gjør den pos. $(= 5)$

f voksende på $[-2, \to)$

Riktig svar: B

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13. $$\lim_{x\to\infty} \frac{x^2-x+1}{xe^x-1} \underset{\uparrow \atop \text{“}\frac{\infty}{\infty}\text{”}}{=} \lim_{x\to\infty} \frac{2x-1}{1\cdot e^x+xe^x} \underset{\uparrow \atop \text{“}\frac{\infty}{\infty}\text{”}}{=} \lim_{x\to\infty} \frac{2}{e^x+1\cdot e^x+x\cdot e^x} \underset{\uparrow \atop \text{“}\frac{2}{\infty}\text{”}}{=} \underline{\underline{0}}$$

Riktig svar: (B) 0

14. $f=(x^2-6x-15)e^x$

$f'=(2x-6)e^x+(x^2-6x-15)e^x$

$\quad=(x^2-4x-21)e^x$

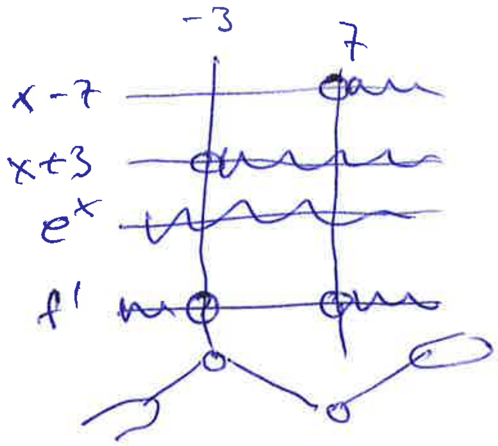
$x^2-4x-21=0$

$x=\dfrac{4\pm\sqrt{16-4(-21)}}{2}$

$\quad=\dfrac{4\pm10}{2}=7,-3$

$\Downarrow$

$x^2-4x-21=(x-7)(x+3)$



$\underline{x=-3}$: $f(-3)=12e^{-3}$
lokalt maks

$\underline{x\to\infty}$: $\lim_{x\to\infty} f(x) = \text{“}\infty\cdot\infty\text{”} = \underline{\infty}$

ikke noe (globalt) maks

$\underline{x=7}$: $f(-7)=-8e^7$
lokalt min

$\underline{x\to-\infty}$: $$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (x^2-6x-15)e^x \underset{\uparrow \atop \text{“}\infty\cdot 0\text{”}}{=} \lim_{x\to-\infty} \frac{x^2-6x-15}{e^{-x}} \underset{\uparrow \atop \text{“}\frac{\infty}{\infty}\text{”}}{=}$$

$$= \lim_{x\to-\infty} \frac{2x-6}{-e^{-x}} \underset{\text{“}\frac{-\infty}{-\infty}\text{”}}{=} \lim_{x\to-\infty} \frac{2}{e^{-x}} \underset{\uparrow \atop \text{“}\frac{2}{\infty}\text{”}}{=} 0 > f(7)$$

$x=7$ globalt min:
$f(7)=-8e^7$
minste verdi

Riktig svar: (C) Minimum med neg. verdi $\underline{-8e^7<0}$

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15. $f=(x+2)\sqrt{x+1}\,,\quad x>-1$

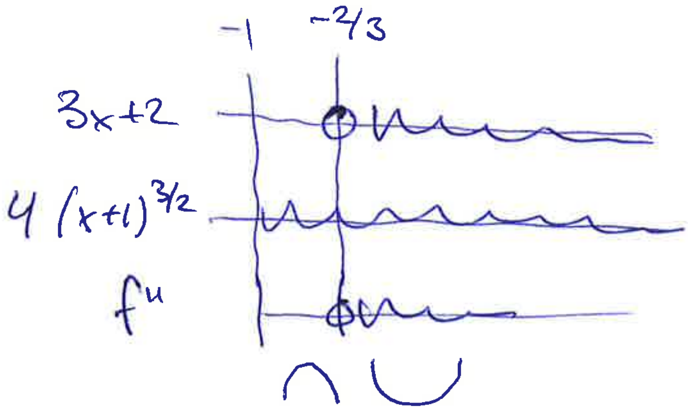
$$f' = 1\cdot\sqrt{x+1} + (x+2)\cdot\frac{1}{2\sqrt{x+1}}\cdot 1$$

$$= \sqrt{x+1} + \frac{x+2}{2\sqrt{x+1}} = \frac{2\sqrt{x+1}\cdot\sqrt{x+1}}{2\sqrt{x+1}} + \frac{x+2}{2\sqrt{x+1}} = \frac{2(x+1)+x+2}{\sqrt{x+1}}$$

$$= \frac{3x+4}{2\sqrt{x+1}}$$

$$f'' = \frac{3\sqrt{x+1}\cdot 2 - (3x+4)\cdot\frac{1}{2\sqrt{x+1}}\cdot 1}{4(x+1)} \quad \begin{matrix}\cdot 2\sqrt{x+1}\\ \cdot 2\sqrt{x+1}\end{matrix}$$

$$= \frac{6(x+1)-(3x+4)}{4(x+1)\sqrt{x+1}} = \frac{3x+2}{4(x+1)\sqrt{x+1}} = \underline{\frac{3x+2}{4(x+1)^{3/2}}}$$



Vendepkt: $x = \underline{\underline{-2/3}}$

<u>Riktig svar:</u> D Vendepkt $a = -2/3 < 0$