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Eksamen MET11802 04/2015 Løsning

1. $f(x)=x^3+5,\ x\in[-1,1]$

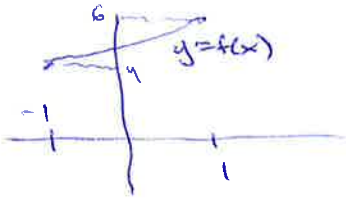
$f'(x)=3x^2\geq 0$ f voksende

$f(-1)=4$

$f(1)=6$

$V_f=[4,6]$

$D_{f^{-1}}=[4,6]$



(B)

2. $g(x,y)=e^{\ln(xy)}$

$=xy$

$g'_x=y \quad g''_{xy}=1$

flere variable → vårens pensum

(A)

3. $(x^2-x-5)^2=1$

$x^2-x-5=\pm 1$

$x^2-x-6=0$ eller $x^2-x-4=0$

$x=\frac{1\pm\sqrt{1+24}}{2}$ $\qquad$ $x=\frac{1\pm\sqrt{1+16}}{2}$

$=\frac{1\pm 5}{2}$ $\qquad$ $=\frac{1\pm\sqrt{17}}{2}$

pos. løsn $\qquad$ pos. løsn

$x_1=3=a$ $\qquad$ $x_2=\frac{1+\sqrt{17}}{2}=b$ $\qquad$ $x_1 x_2 = \frac{3(1+\sqrt{17})}{2}$

$\quad ab$

(D)

4. $g(x)=x\ln x$

$g'(x)=1\cdot\ln x+x\cdot\frac{1}{x}=\ln x+1$

$El_x\, g(x)=\frac{x}{g(x)}\cdot g'(x)$

$=\frac{\cancel{x}\cdot(\ln x+1)}{\cancel{x}\cdot\ln x}=\frac{\ln x+1}{\ln x}$

$=1+\frac{1}{\ln x}$

(A)

5. $f(x,y) = x^3+y^3$ når $x^2+y^2=1$

flere variable → værens param

$L = x^3+y^3-\lambda(x^2+y^2)$

$L'_x = 3x^2-\lambda\cdot 2x = 0 \qquad x(3x-2\lambda)=0$

$L'_y = 3y^2-\lambda\cdot 2y=0 \qquad y(3y-2\lambda)=0$

$x^2+y^2=1 \qquad x^2+y^2=1$

$x=0$:
$y=\pm 1$
$\lambda = \frac{3y}{2} = \pm\frac{3}{2}$
$(0,\pm 1;\ \pm 3/2)$
$f=\pm 1$

$y=0$:
$x=\pm 1$
$\lambda = \frac{3x}{2} = \pm 3/2$
$(\pm 1, 0;\ \pm 3/2)$
$f=\pm 1$

$x,y\neq 0$:
$\lambda = \frac{3x}{2} = \frac{3y}{2}$
$\Rightarrow x=y$
$\Rightarrow 2x^2=1$
$x^2 = 1/2$
$x=y=\pm\sqrt{1/2}$
$\lambda = \pm\sqrt{1/2}\cdot 3/2$
$(\pm\sqrt{1/2}, \pm\sqrt{1/2};\ \pm\frac{3}{2}\sqrt{1/2})$
$f = \sqrt{1/2}\cdot(\pm 1) = \pm\sqrt{2}$

Ingen andre kandidat pkt.
degenerert bibetingelse: $2x=2y=0$
$x=y=0$ passer ikke i $x^2+y^2=1$

Side $x^2+y^2=1$ (en sirkel) er begrenset, er

$V_1 = \max = 1$
$V_2 = \min = -1$
$\Big\}\ V_1\cdot V_2 = -1$ D

6. $g(x) = e^{-x^2+x}$

$g' = e^{-x^2+x}\cdot(-2x+1)$

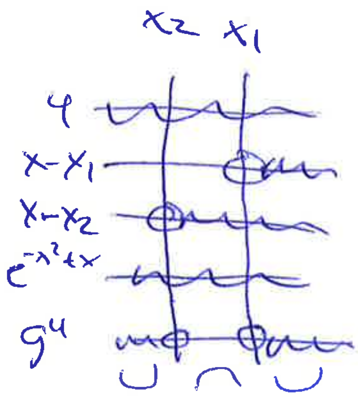
$g'' = e^{-x^2+x}\cdot(-2x+1)^2 + e^{-x^2+x}(-2)$

$= \left((-2x+1)^2-2\right)e^{-x^2+x}$

$= (4x^2-4x-1)\,e^{-x^2+x}$

$= 4(x-x_1)(x-x_2)\,e^{-x^2+x}$

$4x^2-4x-1=0$
$x = \frac{4\pm\sqrt{16+16}}{2\cdot 4}$
$= \frac{1}{2}\pm\frac{1}{2}\sqrt{2}$
$x_1 = \frac{1}{2}+\sqrt{2}\cdot\frac{1}{2}$
$x_2 = \frac{1}{2}-\sqrt{2}\cdot\frac{1}{2}$



$x=x_1,\ x=x_2$
vendepkt.

$x_1\cdot x_2 = (\frac{1}{2}+\frac{1}{2}\sqrt{2})(\frac{1}{2}-\frac{1}{2}\sqrt{2})$
$= 1/4 - 2/4 = -1/4$ B

Dvs: $x_1 \cdot x_2 = \left(\frac{1}{2}+\frac{1}{2}\sqrt{2}\right)\left(\frac{1}{2}-\frac{1}{2}\sqrt{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\sqrt{2}\right)^2$

$= 1/4 - 1/4 \cdot 2 = \frac{1}{4} - \frac{2}{4} = -\frac{1}{4}$

7. $M = \lim_{x \to 0^+} \frac{\ln x}{x + \ln x} \underset{\text{"}\frac{-\infty}{-\infty}\text{"}}{=} \lim_{x \to 0^+} \frac{1/x}{1+1/x} = \lim_{x \to 0^+} \frac{1}{x+1} = \frac{1}{1} = 1$

$N = \lim_{x \to \infty} \frac{\ln x}{x + \ln x} \underset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x \to \infty} \frac{1/x}{1+1/x} = \frac{0}{1+0} = 0$

C

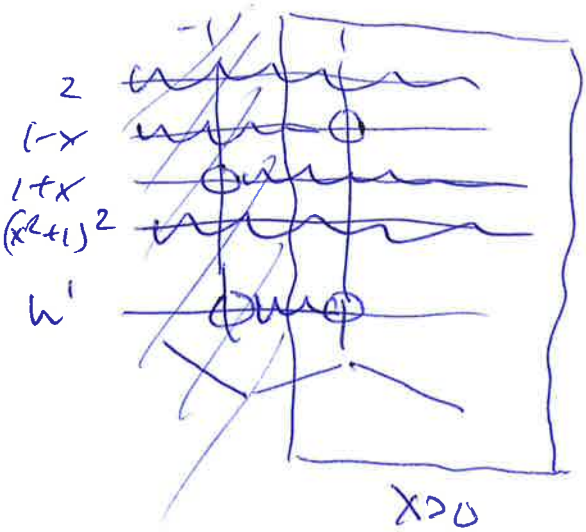
$N + M = 1 + 0 = 1$

8. $K(x) = \ln(x^2+1)$

$K'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$ grensekostnad

Maks grensekostnad $= \max h(x) = \frac{2x}{x^2+1}$

$h' = \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2} = \frac{2(1-x)(1+x)}{(x^2+1)^2}$



X>0

$x>0$: max for $x=1$ C

$h(1) = \frac{2}{2} = 1$

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9. $f(x,y) = y e^{x^2+y^2}$ med $x(t) = t^3$, $y(t) = \sqrt{t}$

flere variable → verstes person

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$= y e^{x^2+y^2} \cdot 2x \cdot 3t^2 + (1 \cdot e^{x^2+y^2} + y \cdot e^{x^2+y^2} \cdot 2y) \cdot \frac{1}{2\sqrt{t}}$$

$$= \sqrt{t}\, e^{t^6+t} \cdot 2t^3 \cdot 3t^2 + e^{t^6+t} \cdot \frac{1}{2\sqrt{t}} \cdot (1+2t)$$

$$= e^{t^6+t} \cdot \frac{1}{2\sqrt{t}} \left(1 + 2t + 2\sqrt{t} \cdot \sqrt{t} \cdot 3t^2 \cdot 2t^3\right)$$

$$= \frac{1+2t+12t^6}{2\sqrt{t}} e^{t^6+t}$$

A

10. $(x^5 + x^3 + x + 1) : (x^2+1) = x^3$

$$-(x^5+x^3)$$

$$x+1$$

$$\rightarrow x^3 + \frac{x+1}{x^2+1}$$

A

11. $x \ln y + y \ln x + y^2 = 1$ i $(1,1)$:

implisitt derivasjon: $1 \cdot \ln y + x \cdot \frac{1}{y} \cdot y' + y' \cdot \ln x + y \cdot \frac{1}{x} + 2y \cdot y' = 0$

$$\left(\frac{x}{y} + \ln x + 2y\right) y' = -\ln y - \frac{y}{x}$$

$$y' = -\frac{\ln y + \frac{y}{x}}{\ln x + \frac{x}{y} + 2y} = -\frac{1}{3}$$

$(x,y) = (1,1)$

$$y - 1 = -\frac{1}{3}(x-1) \qquad y = -\frac{1}{3}x + \frac{1}{3} + 1 = -\frac{1}{3}x + \frac{4}{3}$$

C

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12. $f(x,y) = (x^2+y^2)e^{xy}$

flere variabler
→ vårens pensum

$f'_x = 2x \cdot e^{xy} + (x^2+y^2)e^{xy} \cdot y$
$= (2x + y(x^2+y^2))e^{xy} = 0$
$f'_y = (2y + x(x^2+y^2))e^{xy} = 0$

$x, y \neq 0$: $2x + y(x^2+y^2) = 0 \Rightarrow x^2+y^2 = -\frac{2x}{y}$
$2y + x(x^2+y^2) = 0 \quad x^2+y^2 = -\frac{2y}{x}$

$-\frac{2x}{y} = -\frac{2y}{x}$
$-2x^2 = -2y^2$
$x^2 = y^2$
$x = \pm y$

$x = y$: $2x + x \cdot 2x^2 = 0$
$2x(1+x^2) = 0$
$2x \neq 0, \quad 1+x^2 \neq 0$
ingen løsn.

$x = -y$: $2x - x(2x^2) = 0$
$2x(1-x^2) = 0$
$x = \pm 1$ → $x = 1, y = -1$
$x = -1, y = 1$

$x = 0$: $y = 0$
$y = 0$: $x = 0$ → $x = 0, y = 0$

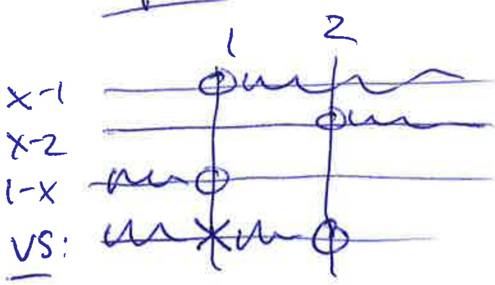
3 stasjonære pkt (A)

13. $\frac{x^2-1}{1-x} \geq -3$

$\frac{x^2-1}{1-x} + 3 \cdot \frac{1-x}{1-x} \geq 0$

$\frac{x^2-1+3-3x}{1-x} \geq 0$

$\frac{x^2-3x+2}{1-x} \geq 0 \qquad \frac{(x-1)(x-2)}{(1-x)} \geq 0$



Løsn:
$(-\infty, 1) \cup (1, 2]$ (B)

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14.

$$g(x) = e^{3x+2}$$

$$g'(x) = e^{3x+2} \cdot 3$$

$$\frac{g'(x)}{g(x)} = \frac{3e^{3x+2}}{e^{3x+2}} = 3$$

C

15.

$$K(x) = x^2 + 6x + 1$$

$$P(x) = 20 - x$$

$$\pi(x) = \underbrace{x \cdot p(x)}_{\text{inntekt}} - \underbrace{K(x)}_{\text{kostnad}} = x \cdot (20 - x) - (x^2 + 6x + 1)$$

$$= 20x - x^2 - x^2 - 6x - 1$$

$$= -2x^2 + 14x - 1$$

$$\pi' = -4x + 14 = 0$$

$$x = 14/4 = 7/2$$

$\pi'' = -4 < 0 \Rightarrow x = 7/2$ er globalt maks.

A