	Emne	Lærebok	Oppgaver
1	Repetisjon og oppgaveregning		[Ark] Oppgave 7
2	Funksjoner i to variabler	[E] 7.1	[E] 7.1.1 - 7.1.2
3	Grafer og nivåkurver	[E] 7.1	[E] 7.1.3 - 7.1.4

1 Repetisjon:

A $m\times n$-matrise

A invertibel hvis $n=m$ og $|A|\neq 0$, med

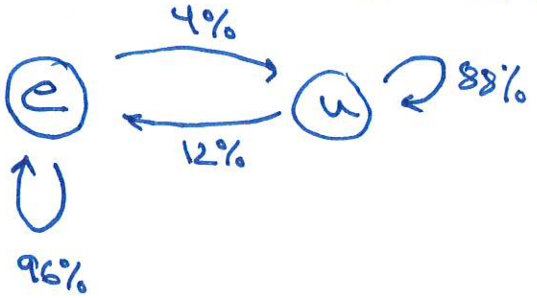
$$A^{-1} = \frac{1}{|A|}\cdot\begin{pmatrix} C_{11} & C_{12} & \cdots \\ C_{21} & C_{22} & \cdots \\ \vdots & \vdots & \end{pmatrix}^T$$

A ikke invertibel hvis $m\neq n$ eller $|A|=0$.

Oppgave 7:

e_t: i arbeid (andel)

u_t: arbeidsledig $\quad e_t+u_t=1$



a) $e_1 = 0.96\cdot e_0 + 0.12\cdot u_0$

$u_1 = 0.04\, e_0 + 0.88\, u_0$

$$\underline{v_1} = \underbrace{\begin{pmatrix} 0.96 & 0.12 \\ 0.04 & 0.88 \end{pmatrix}}_{A}\underline{v_0}$$

$\underline{v_t} = \begin{pmatrix} e_t \\ u_t \end{pmatrix}$

b) $u_0 = 0.15$ $\quad \underline{v_0} = \begin{pmatrix} 0.85 \\ 0.15 \end{pmatrix}$ $\quad \underline{v_1} = A\cdot \underline{v_0}$ $\quad \underline{v_2} = A\cdot\underline{v_1} = A\cdot(A\underline{v_0}) = A^2\cdot\underline{v_0}$

$$\underline{v_2} = \begin{pmatrix} 0.96 & 0.12 \\ 0.04 & 0.88 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.12 \\ 0.04 & 0.88 \end{pmatrix}\cdot\begin{pmatrix} 0.85 \\ 0.15 \end{pmatrix} \simeq \begin{pmatrix} 0.82 \\ 0.18 \end{pmatrix}$$

$u_2 \hat{=} 18\%$

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c) $\begin{pmatrix} e_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} e_t \\ u_t \end{pmatrix} : \quad A \cdot \begin{pmatrix} e_t \\ u_t \end{pmatrix} = \begin{pmatrix} e_t \\ u_t \end{pmatrix}$

$$\begin{pmatrix} 0.96 e_t + 0.12 u_t \\ 0.04 e_t + 0.88 u_t \end{pmatrix} = \begin{pmatrix} 0.96 & 0.12 \\ 0.04 & 0.88 \end{pmatrix} \begin{pmatrix} e_t \\ u_t \end{pmatrix} = \begin{pmatrix} e_t \\ u_t \end{pmatrix}$$

$$\begin{aligned} 0.96 e_t + 0.12 u_t &= e_t \\ 0.04 e_t + 0.88 u_t &= u_t \end{aligned} \qquad \begin{aligned} -0.04 e_t + 0.12 u_t &= 0 \\ 0.04 e_t - 0.12 u_t &= 0 \end{aligned}$$

$$\begin{array}{c} \left(\begin{array}{cc|c} -0.04 & 0.12 & 0 \\ 0 & 0 & 0 \end{array}\right) \\ \begin{array}{cc} \uparrow & \uparrow \\ e_t & u_t \end{array} \end{array} \leftarrow \left[\left(\begin{array}{cc|c} -0.04 & 0.12 & 0 \\ 0.04 & -0.12 & 0 \end{array}\right)\right]^1$$

u_t fri, $\quad -0.04\, \underline{e_t} + 0.12 u_t = 0$

$$\frac{\cancel{-0.04} e_t}{\cancel{-0.04}} = \frac{-0.12 u_t}{-0.04} \qquad e_t = 3 u_t$$

<u>Mesk:</u> $e_t + u_t = 1$ $\qquad e_t = 3/4 = 0.75$ (likevekts-

$3u_t + u_t = 1 \quad 4u_t = 1 \quad u_t = 1/4 = 0.25$ tilstand

d) $\begin{pmatrix} e_{60} \\ u_{60} \end{pmatrix} = A^{60} \cdot \begin{pmatrix} e_0 \\ u_0 \end{pmatrix} \simeq \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix}$

$\underline{v}_{60} = \begin{pmatrix} e_{60} \\ u_{60} \end{pmatrix}$, $\quad \underline{v}_0 = \begin{pmatrix} e_0 \\ u_0 \end{pmatrix} = \begin{pmatrix} 0.85 \\ 0.15 \end{pmatrix}$

② Funksjon i flere variabler   flere = to

Eks:

$f(x,y) = 2x - 3y + 1$ lineær fn.
$f(x,y) = x^2 + y^2$ kvadr. fn.
} polynom-fn.

$f(x,y) = \dfrac{x+y}{x-y}$ rasjonal fn.

$f(x,y) = 104\, x^{1.2}\, y^{1.7}$ Cobb-douglas fn.
$= 104\, x^{6/5}\, y^{17/10}$
$= 104\, \sqrt[5]{x^6} \cdot \sqrt[10]{y^{17}}$

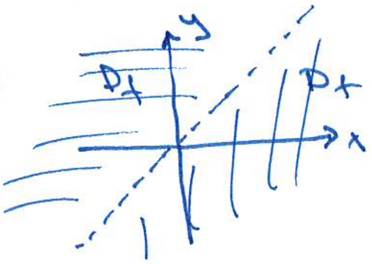
$f(x,y) = xy\,e^{x+y}$

$f(x,y) = \ln(x^2+y^2+1)$

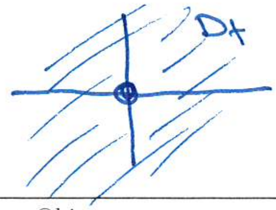
Det. område: Hvis $\underbrace{D_f = \text{det. område for } f}$ ikke er spesifisert, så antar vi at D_f er alle tallpar (x,y) slik at $f(x,y)$ gir mening.

Eks: $f(x,y) = x^2 + y^2$ $D_f =$ alle tallpar $(x,y) = \mathbb{R}^2$

$f(x,y) = \dfrac{x+y}{x-y}$ $D_f:\ x - y \neq 0$
$x \neq y$



$f(x,y) = \ln(x^2+y^2)$, $D_f: (x,y) \neq (0,0)$



Eks: $f(x,y) = 104\, x^{1.2} y^{1.7}$, $x \geq 0,\ y \geq 0$ ← spesifisert basert på anvendelse

$f(x,y) = 104\, x^{1.2} y^{1.7}$

$= 104 \sqrt[5]{x^6}\ \sqrt[10]{y^{17}}$ $\underline{D_f: x,y \geq 0}$

$x^{1.2} \cdot x^{1.3} = x^{2.5}$

$\sqrt[5]{x^6}$ $\sqrt[10]{x^{13}}$ $\sqrt{x^5}$

x vilk. $x \geq 0$ $x \geq 0$

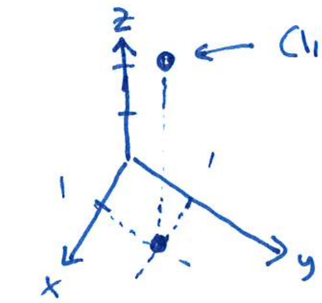
Kan oppstå problemer om x,y negative, velger ofte $\underline{x,y \geq 0}$ når eksponentene ikke er hele tall

② Grafen til f

Eks: $f(x,y) = x^2 + y^2 + 1$

x	y	$f(x,y) = z$
0	0	1
1	0	2
1	1	3

input output



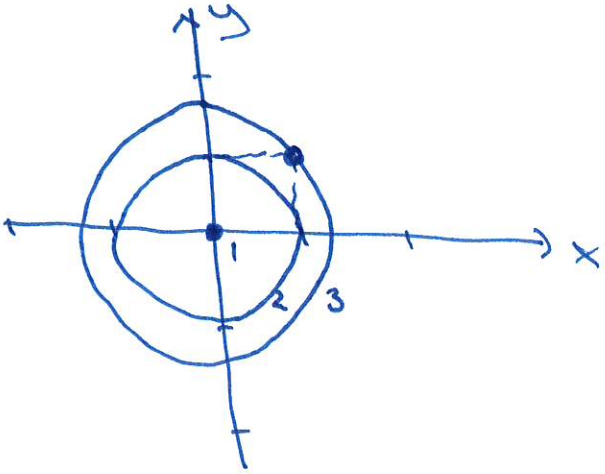
Grafen til f = alle pkt $\underline{(x,y,z)}$ slik at (x,y) er i D_f, $z = f(x,y)$, representert i xyz-koordinatsys.

Nivåkurver: $f(x,y) = c$ for en konstant c

Eks: $c = 3$ ← alle pkt i høyde 3

$x^2 + y^2 + 1 = 3$

$\underline{x^2 + y^2 = 2}$ ← sirkel m/sentr $(0,0)$, $r = \sqrt{2}$



$f(x,y)=3$: pkt i høyde 3

$f(x,y)=2$: — " — 2

$x^2+y^2+1=2$

$x^2+y^2=1$

$f(x,y)=1$: $x^2+y^2+1=1$

$x^2+y^2=0$

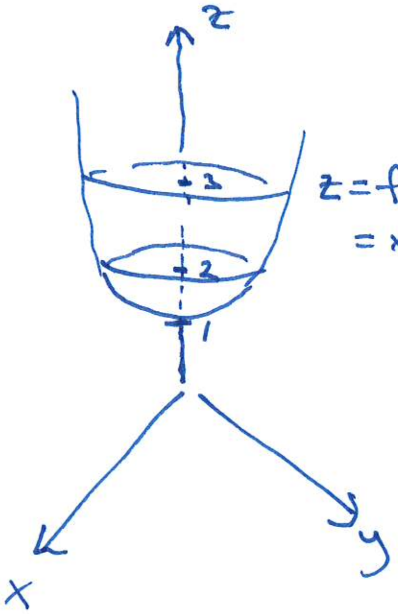
$(0,0)$

$f(x,y)=c$, $c<1$:

$x^2+y^2+1=c$

$x^2+y^2=c-1<0$

ingen pkt



$z=f(x,y)$
$=x^2+y^2+1$

$f(x,y)=c$, $c>3$

$x^2+y^2+1=c$

$x^2+y^2=c-1>2$

sirkel m/ $r=\sqrt{c-1}$

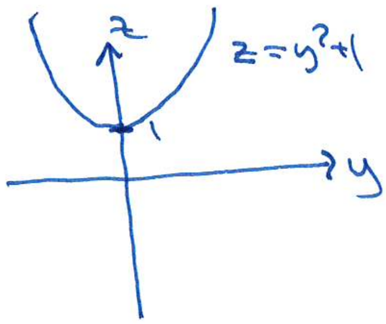
Nivåkurve: $z=c \iff f(x,y)=c$ horisontalt kutt gjennom grafen

Andre kutt: $x=0 \iff f(0,y)=z$ vertikalt kutt $x=0$

Eks: $f(x,y)=x^2+y^2+1$

$x=0$: $f(0,y)=0^2+y^2+1$

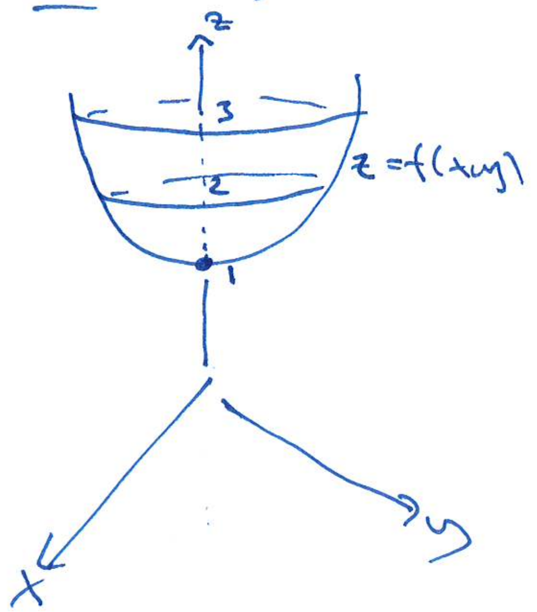
$z=y^2+1$



$z=y^2+1$

Verdimengden til f: V_f = mengden av alle mulige funksjonsverdier $f(x,y)$

Eks: $f(x,y) = x^2+y^2+1$



$V_f = [1, \to)$

$f_{max} = \infty$

$f_{min} = 1$

Nivåkurver:

$z = c$ er i V_f

$\Updownarrow$

$f(x,y) = c$ inneholder minst ett pkt.

Eks: $f(x,y) = x^2+y^2+1$

Nivåkurver: $x^2+y^2+1 = c$

$x^2+y^2 = c-1$

har løsninger for $c \geq 1$

har ikke løsn. $c < 1$

Eks: $f(x,y) = 2x-3y+1$ lin. fn. $D_f = \mathbb{R}^2$

grafen til f: et plan (uten krumning)

Nivåkurver: $f(x,y) = c$

$2x-3y+1 = c$ ← rett linje

$$\frac{-3y}{-3} = \frac{c-1-2x}{-3}$$

$$y = \frac{2}{3}x + \frac{1-c}{3}$$

$V_f = (-\infty, \infty)$