	Emne	Lærebok	Oppgaver
1	Matriseregning og matrisemultiplikasjon	[E] 6.6	[E] 6.6.1 - 6.6.5
2	Noen regneregler for matriser	[E] 6.6	

[E] 6.8
Pensum fram til ortogonal projeksjon

Vektorer og vektorregning:

- lineærkombinasjoner, span, vektorlikninger
- lengden av en vektor, ortogonalitet, indreprodukt

$$x \cdot \begin{pmatrix} 80-60 \\ 100-40 \\ 40-60 \end{pmatrix} + y \cdot \begin{pmatrix} -- \\ - \\ -- \end{pmatrix} + z \cdot \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}$$

① Matriser og matriseregning

Defn En $m \times n$-matrise er en rektangulær tabell med tall/tallverdier som har m rader og n kolonner

tallet i rad 1, kolonne 2

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$ $m \times n$-matrise

Eks: $A = \begin{pmatrix} 1 & 4 & 2 \\ 0 & 7 & -1 \end{pmatrix}$ 2×3-matrise $\quad B = \begin{pmatrix} 1 & 2 & a \\ a & -1 & 0 \\ 1 & a & 4 \end{pmatrix}$ 3×3-matrise

A kalles kvadratisk hvis $m=n$.

Regneoperasjoner:

- addisjon / subtraksjon: $A \pm B$ — definert om A og B har samme størrelse

Eks: $\begin{pmatrix} 1 & 7 & -1 \\ 0 & 4 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 4 \\ 3 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 1+2 & 7+0 & -1+4 \\ 0+3 & 4+(-1) & 2+3 \end{pmatrix}$

$$= \underline{\begin{pmatrix} 3 & 7 & 3 \\ 3 & 3 & 5 \end{pmatrix}}$$

- skalarmultiplikasjon: $r \cdot A$ (r skalar = tall)

Eks: $$2 \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix} = \underline{\begin{pmatrix} 2 & 2 & 2 \\ 2 & 4 & 8 \\ 2 & 6 & 18 \end{pmatrix}}$$

- matrisemultiplikasjon: $\underset{m\times n}{A} \cdot \underset{n\times p}{B}$ — definert når # kolonner i A = # rader i B

$$\underbrace{\qquad}_{m\times p} \quad AB$$

produkt:

rader = # rader i A

kolonner = # kolonner i B

Eks:

$$\underset{2\times 2}{\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}} \cdot \underset{2\times 1}{\begin{pmatrix} 7 \\ 3 \end{pmatrix}} = \underset{2\times 1}{\begin{pmatrix} \bullet \\ \bullet \end{pmatrix}}$$

13

17

$2\times 2 \;\; 2\times 1$: $\underbrace{2 \quad 2}_{=}$

$$= \underline{\underline{\begin{pmatrix} 13 \\ 17 \end{pmatrix}}}$$

$\begin{pmatrix} 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 7 \\ 3 \end{pmatrix} \rightsquigarrow 1\cdot 7 + 2\cdot 3 = \underline{13}$

$\begin{pmatrix} 2 & 1 \end{pmatrix} \quad \begin{pmatrix} 7 \\ 3 \end{pmatrix} \rightsquigarrow 2\cdot 7 + 1\cdot 3 = \underline{17}$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 3 \end{pmatrix} = 1\cdot 7 + 2\cdot 3 = 13$$

prikkprodukt

<u>Eks:</u>

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 3 & 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 5 \\ 3 & 4 \end{pmatrix}}}$$

$$\underbrace{2\times 2 = 2\times 2} \qquad 2\times 2$$

$$\begin{array}{c|c} & \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \\ \hline \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} & \begin{pmatrix} 3 & 5 \\ 3 & 4 \end{pmatrix} \end{array}$$

<u>Merk:</u> $A \cdot B \neq B \cdot A$

$AB \neq BA$

<u>Eks:</u>

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 3 \\ 5 & 4 \end{pmatrix}}}$$

$$\underbrace{2\times 2 = 2\times 2}$$

<u>Eks:</u>

$$\begin{aligned} x + y + z + w &= 4 \\ x - y + 2z - w &= 1 \\ 2x + 3y - z - w &= 3 \end{aligned}$$

3×4 lin. system $\rightsquigarrow$

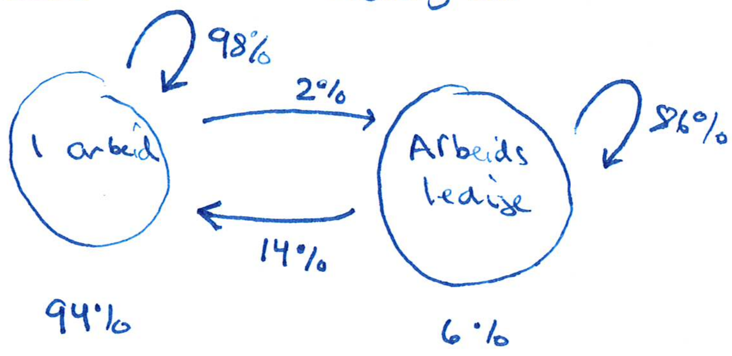
<u>Matriseform:</u> $A \cdot \underline{x} = \underline{b}$

<u>Gauss:</u> $(A \mid \underline{b})$

$$\underset{\substack{\shortparallel \\ A \\ \text{koeff.matrisen}}}{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & -1 \\ 2 & 3 & -1 & -1 \end{pmatrix}} \qquad \underline{x} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \qquad \underset{\substack{\shortparallel \\ \underline{b}}}{\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}}$$

$$\underbrace{\underset{3\times 4}{A} \cdot \underset{4\times 1}{\underline{x}}} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & -1 \\ 2 & 3 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x + y + z + w \\ x - y + 2z - w \\ 2x + 3y - z - w \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} = \underline{b}$$

Eks: Arbeidsledighet



Tilstandsvektor: $\underline{v_0} = \begin{pmatrix} 0.94 \\ 0.06 \end{pmatrix} \begin{array}{l} \leftarrow e_0 \\ \leftarrow u_0 \end{array}$

Endringsmatrise
$$A = \begin{array}{cc|c} e & u & \\ \hline \multicolumn{2}{c|}{\begin{pmatrix} 0.98 & 0.14 \\ 0.02 & 0.86 \end{pmatrix}} & \begin{array}{c} e \\ u \end{array} \end{array}$$

$$\underline{v_1} = A \cdot \underline{v_0} = \begin{pmatrix} 0.98 & 0.14 \\ 0.02 & 0.86 \end{pmatrix} \cdot \begin{pmatrix} 0.94 \\ 0.06 \end{pmatrix}$$

$$= \begin{pmatrix} 0.98 \cdot 0.94 + 0.14 \cdot 0.06 \\ 0.02 \cdot 0.94 + 0.86 \cdot 0.06 \end{pmatrix} \begin{array}{l} \leftarrow e_1 \\ \leftarrow u_1 \end{array}$$

$$\underline{v_2} = A \cdot \underline{v_1} = A \cdot (A \cdot \underline{v_0}) = A^2 \cdot \underline{v_0}$$

$$\vdots$$

$$\underline{v_n} = \ldots = A^n \cdot \underline{v_0}$$

$$A^2 = \begin{pmatrix} 0.98 & 0.14 \\ 0.02 & 0.86 \end{pmatrix} \cdot \begin{pmatrix} 0.98 & 0.14 \\ 0.02 & 0.86 \end{pmatrix}$$

$$= \begin{pmatrix} 0.98^2 + 0.14 \cdot 0.02 & --- \\ 0.02 \cdot 0.98 + 0.86 \cdot 0.02 & --- \end{pmatrix}$$

Potenser av matriser:

A : $n \times n$-matrise (kvadratisk)

$$\underset{n \times n}{A^2} = \underset{n \times n}{A \cdot A}$$

$$A^3 = (A \cdot A) A = A^2 \cdot A$$

$$\vdots$$

$$A^n = \underbrace{A \cdot A \cdot \ldots \cdot A}_{n \text{ faktorer}}$$

Transponering: $\underset{m \times n}{A} \dashrightarrow \underset{n \times m}{A^T}$

Eks: $A = \begin{pmatrix} 1 & 3 & 7 \\ 0 & 1 & 4 \end{pmatrix}$ 2×3 $\qquad A^T = \begin{pmatrix} 1 & 0 \\ 3 & 1 \\ 7 & 4 \end{pmatrix}$ 3×2

$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$ 3×3 $\qquad A^T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ 3×3

$A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & -1 & 7 \\ 5 & 6 & 8 \end{pmatrix}$ $\qquad A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 6 \\ 4 & 7 & 8 \end{pmatrix}$

Defn: A kalles symmetrisk hvis $A^T = A$.

Eks: $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 7 \end{pmatrix}$ er symmetrisk $A^T = A$

Fakta: i) $(A \cdot B)^T = B^T \cdot A^T$

ii) $\underline{v} \cdot \underline{w} = \underline{v}^T \cdot \underline{w}$

n-vektorer

prikkprodukt av vektorer

$1 \times n$

matrise-multiplikasjon

$n \times 1$

$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = v_1 w_1 + v_2 \cdot w_2 + \dots + v_n \cdot w_n$

$(v_1 \; v_2 \dots v_n) \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = (v_1 w_1 + v_2 \cdot w_2 + \dots + v_n \cdot w_n)$

Identitetsmatrise:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (I_2)$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (I_3)$$

Egenskap:

$$\underset{m\times n}{A} \cdot \underset{n\times n}{I} = A$$

$$\underset{m\times m}{I} \cdot \underset{m\times n}{A} = A$$

for alle $m\times n$-matriser A

Eks:

$$\underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}}_{A} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{I} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}}_{A}$$

Fakta:

$$\boxed{\det(AB) = \det(A) \cdot \det(B)}$$