	Emne	Lærebok	Oppgaver
1	Repetisjon og oppgavegjennomgang		
2	Indreprodukt og ortogonalitet	[E] 6.8	[E] 6.8.1 - 6.8.3

1 Repetisjon: Vektorer

$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$: $\|\underline{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

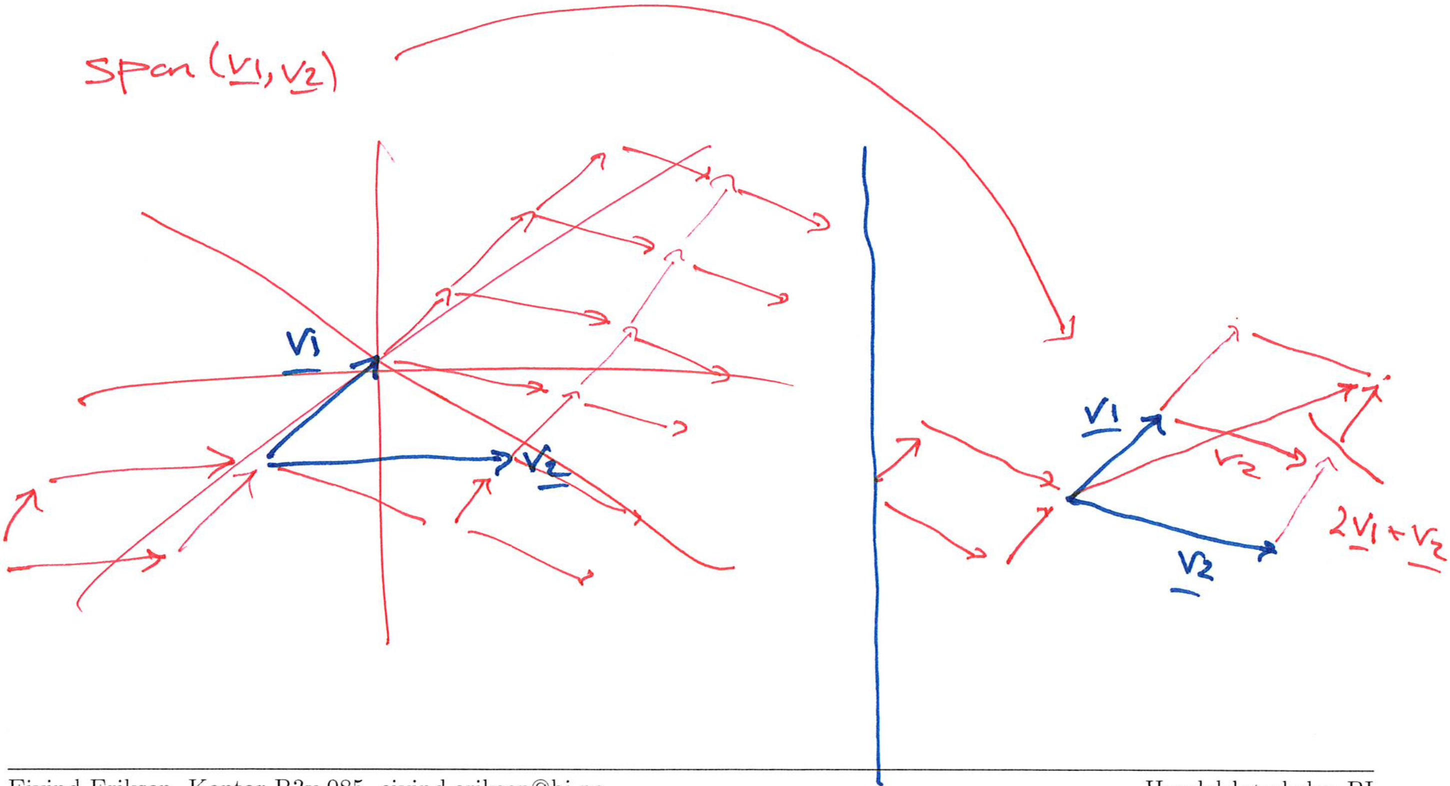
lengden til $\underline{v}$

$\underline{v}_1, \underline{v}_2, \dots, \underline{v}_r$: $c_1 \cdot \underline{v}_1 + c_2 \cdot \underline{v}_2 + \dots + c_r \cdot \underline{v}_r$ ($c_1, c_2, \dots, c_r$ tall)

lineærkomb. av $\underline{v}_1, \dots, \underline{v}_r$

$\text{span}(\underline{v}_1, \underline{v}_2, \dots, \underline{v}_r)$ = mengden av alle lin. komb. av $\underline{v}_1, \dots, \underline{v}_r$

span$(\underline{v}_1, \underline{v}_2)$



Vektorlikning: $c_1 \cdot \underline{v_1} + c_2 \cdot \underline{v_2} + \dots + c_r \cdot \underline{v_r} = \underline{w}$

$c_1 = x_1, \quad c_2 = x_2, \quad c_r = x_r$

= lineær system med utvidet matrise $\longrightarrow$

$$\left(\begin{array}{c:c:c:c|c} \underline{v_1} & \underline{v_2} & \dots & \underline{v_r} & \underline{w} \end{array} \right)$$

Oppgaveark 35, 9

a) $\underbrace{60x + 75y + 320z}_{\text{kostpris for porteføljen}} = \underbrace{400.000}_{\text{tilgjengelig for investering}}$ budsjettbetingelse

b)
$$\begin{aligned} 20x + 5y + 30z &= R_1 \\ 40x - 50y + 180z &= R_2 \\ -20x + 25y - 265z &= R_3 \\ 60x + 75y + 320z &= 400' \end{aligned}$$

$$\left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 40 & -50 & 180 & R_2 \\ -20 & 25 & -265 & R_3 \\ 60 & 75 & 320 & 400' \end{array} \right) \begin{matrix} \\ -2 \\ 1 \\ \cdot 3 \end{matrix}$$

$$\rightarrow \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 30 & -235 & R_3 + R_1 \\ 0 & 60 & 230 & 400' - 3R_1 \end{array} \right) \begin{matrix} \\ \frac{1}{2} \\ 1 \\ \end{matrix} \rightarrow \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 0 & -175 & R_3 + R_1 + \frac{1}{2}(R_2 - 2R_1) \\ 0 & 0 & 350 & \cancel{400' - 3R_1 + R_1} \end{array} \right) \begin{matrix} \\ \\ \\ 2 \end{matrix}$$

$400' - 3R_1 + R_2 - 2R_1 = 400' - 5R_1 + R_2$

$$\rightarrow \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 0 & -175 & R_3 + \frac{1}{2}R_2 \\ 0 & 0 & 0 & \cancel{400' - 2R_1 + 2(R_3 + \frac{1}{2}R_2) = 400' - 2R_1 + R_2 + 2R_3} \end{array} \right)$$

$400' - 5R_1 + R_2 + 2(R_3 + \frac{1}{2}R_2) = 400' - 5R_1 + 2R_2 + 2R_3$

Konklusjon: Mulig å få avkastning (R_1, R_2, R_3)

$$\Updownarrow$$

$400' - 5R_1 + 2R_2 + 2R_3 = 0$, dvs

$$\boxed{5R_1 - 2R_2 - 2R_3 = 400'}$$

b) $R_1 = 50'$, $R_2 = 25'$, $R_3 = -100'$

$5 \cdot 50' - 2 \cdot 25' - 2 \cdot (-100')$

$= 250' - 50' + 200' = 400'$ ok

Ja, det er mulig.

Portefølje:

$20x + 5y + 30z = 50'$ $= 50'$

$-60y + 120z = 25' - 2 \cdot 50' = -75'$

$-175z = -100' + \frac{1}{2} \cdot 25' = -87.5'$

$$\frac{-175z}{-175} = \frac{-87.5'}{-175} \qquad z = 500$$

$$-60y + \underbrace{120 \cdot 500}_{60'} = -75'$$

$-60y = -75' - 60' = -135'$

$$y = \frac{-135'}{-60} = \frac{135'}{60} = 2250$$

$20x + 5(2250) + 30(500) = 50'$

$20x = 50' - 15' - 11.250' = 23.750'$

$$x = \frac{23.750'}{20} = \frac{2375}{2} = 1187.5$$

Konkl:

Portefølje: $x = 500$

$y = 2250$

$z = 1187.5$

c) $R_1 > 0$, $R_2 = R_3 = 0$: $5R_1 = 400'$ $R_1 = \frac{400'}{5} = 80' > 0$

Ja, mulig om $R_1 = 80'$

Portefølje: Sett inn $R_1 = 80'$, $R_2 = R_3 = 0$

Løs for x, y, z

d) (R_1, R_2, R_3) mulig avkastning $\Leftrightarrow$ $5R_1 - 2R_2 - 2R_3 = 400'$

$\underline{R_1 = 100'}$: $500' - 2R_2 - 2R_3 = 400'$

$2R_2 + 2R_3 = 100'$

en av mange mulige løsninger

$\underline{R_2 = 25'}$ $\underline{R_3 = 25'}$

c) Fortsatt: $R_1 = 80'$ $R_2 = R_3 = 0$

Portefølje: $20x + 50y + 30z = 80'$

$-60y + 120z = -2 \cdot 80' = -160'$

$-175z = \frac{1}{2} \cdot 0 + 0 = 0$

$-175z = 0 \Rightarrow \underline{z = 0}$

$-60y + 120 \cdot 0 = -160'$

$-60y = -160'$

$y = \frac{-160'}{-60} = \frac{160.00\cancel{0}}{6\cancel{0}} = \frac{16.000}{6} = \underline{2666\,^2/_3}$

$\simeq \underline{2666{,}67}$

$20x + 50(2666\,^2/_3) + 30(0) = 80'$

$20x = 80' - 50(2666\,^2/_3) = 66.666\,^2/_3$

$x = \frac{66.666\,^2/_3}{20} = \underline{3333\,^1/_3} \simeq 3333{,}33$

② Indre produkt (prikkprodukt, skalarprodukt)

Defn: $\underline{v} \cdot \underline{w} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = v_1 \cdot w_1 + v_2 \cdot w_2 + \dots + v_n \cdot w_n$ (et tall)

Eks: $\underline{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\underline{w} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\underline{v} \cdot \underline{w} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 2 \cdot (-1) + 1 \cdot 3 = \underline{\underline{1}}$

$\underline{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\underline{w} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ $\underline{v} \cdot \underline{w} = 1 \cdot 2 + 1 \cdot (-1) + 2 \cdot 3 = \underline{\underline{7}}$

$\underline{v} \cdot \underline{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \underbrace{1^2 + 1^2 + 2^2}_{\|v\|^2} = 6$

Husk:

$\|\underline{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

$\|\underline{v}\|^2 = v_1^2 + v_2^2 + \dots + v_n^2$

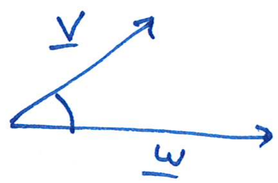
$= \underline{v} \cdot \underline{v}$

Tolkning av prikkprodukt:

$\underline{v} \cdot \underline{w} > 0$ vinkelen mellom $\underline{v}$ og $\underline{w}$ er $< 90°$

$\boxed{\underline{v} \cdot \underline{w} = 0 \quad \text{—— ” ——} \quad = 90°}$

$\underline{v} \cdot \underline{w} < 0$ —— ” —— $> 90°$



Defn: Vektorene $\underline{v}$ og $\underline{w}$ kalles ortogonale hvis $\underline{v} \cdot \underline{w} = 0$

$\Updownarrow$

vinkelen er $90°$

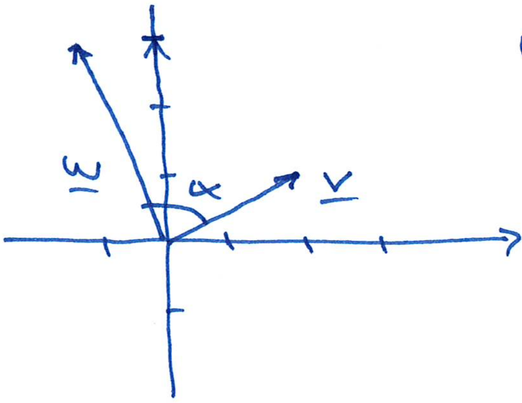
skrivemåte: $\underline{v} \perp \underline{w}$

Eks:

$\underline{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\underline{w} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

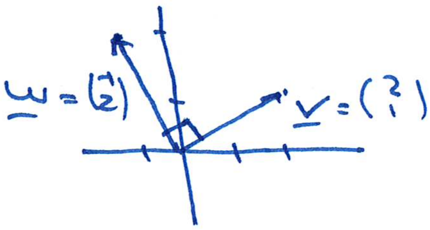
$\underline{v} \cdot \underline{w} = -2+3 = 1$



$\alpha < 90^\circ$

$\underline{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\underline{w} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

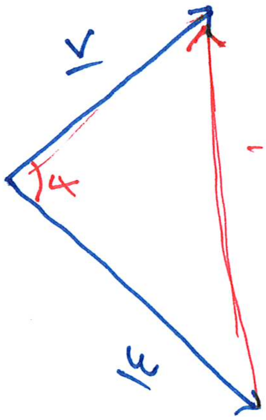


$\underline{v} \cdot \underline{w} = 2 \cdot (-1) + 1 \cdot 2 = 0$

Forklaring:

$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$

$\underline{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$



$\alpha = 90^\circ \Leftrightarrow$ pytagoras' setning holder

$-\underline{w} + \underline{v} = \underline{v} - \underline{w}$

$= \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} - \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix} = \begin{pmatrix} v_1 - w_1 \\ v_2 - w_2 \\ \vdots \\ v_n - w_n \end{pmatrix}$

$\|\underline{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

$\|\underline{w}\| = \sqrt{w_1^2 + w_2^2 + \dots + w_n^2}$

$$\text{katet}^2 + \text{katet}^2 = v_1^2 + v_2^2 + \ldots + v_n^2 + w_1^2 + w_2^2 + \ldots + w_n^2$$

$$\text{hypotenus}^2 = \left\| \begin{pmatrix} v_1 - w_1 \\ v_2 - w_2 \\ \vdots \\ v_n - w_n \end{pmatrix} \right\|^2 = \underline{(v_1-w_1)^2} + \underline{(v_2-w_2)^2} + \ldots + (v_n-w_n)^2$$

$$= v_1^2 - 2v_1w_1 + w_1^2 + v_2^2 - 2v_2w_2 + w_2^2 + \ldots + v_n^2 - 2v_nw_n + w_n^2$$

<u>Pytagoras' setn. holder</u>:

$$\cancel{v_1^2} + \cancel{v_2^2} + \ldots + \cancel{v_n^2} + \cancel{w_1^2} + \cancel{w_2^2} + \ldots + \cancel{w_n^2} = \cancel{v_1^2} \underline{-2v_1w_1} + \cancel{w_1^2} + \cancel{v_2^2} \underline{-2v_2w_2} + \cancel{w_2^2} + \ldots + \cancel{v_n^2} - 2v_nw_n + \cancel{w_n^2}$$

$$0 = -2v_1w_1 - 2v_2w_2 - \ldots - 2v_n \cdot w_n \quad | : (-2)$$

$$v_1w_1 + v_2w_2 + \ldots + v_n \cdot w_n = 0$$

$$\underline{v} \cdot \underline{w} = 0$$