

| Emne | Lærebok | Oppgaver |
|--|---------|-------------------|
| 1 Repetisjon og oppgavegjennomgang | | [Ark] 6b, 7, 8b |
| 2 Antall løsninger av lineære systemer | [E] 6.3 | [E] 6.3.1 - 6.3.8 |
| 3 Gauss-Jordan eliminasjon | [E] 6.2 | [E] 6.2.5 |

① Repetisjon

Lineært system = likningssystem der alle likningene er lineære

Gauss eliminasjon: metode for å løse lineære systemer

- Skriv ned den utvidede matrisen til det lineære systemet
- bruk elementære radoperasjoner til vi får en trappetform
- skriv tilbake til et lineært system, og løs via baklengs substitusjon.

Fakta:

- enhver matrisen kan gjøres om til en trappetform via elementære radoperasjoner
- en trappetform er ikke entydlig
ne løsninger er de samme
og pivotposisjonen er de samme

Ark 31:

$$\underline{6b)} \quad \left(\begin{array}{ccc|c} \textcircled{3} & 4 & 3 & 2 \\ 2 & -1 & 1 & 1 \\ 7 & 2 & 5 & 3 \end{array} \right) \begin{array}{l} \uparrow -2/3 \\ \leftarrow -7/3 \end{array} \rightarrow \left(\begin{array}{ccc|c} \textcircled{3} & 4 & 3 & 2 \\ 0 & -1 - \frac{2}{3} & 1 - \frac{2}{3} & 1 - \frac{2}{3} \\ 0 & 2 - \frac{2}{3} & -2 & 3 - \frac{14}{3} \end{array} \right)$$

=

$$\left(\begin{array}{ccc|c} \textcircled{3} & 4 & 3 & 2 \\ 0 & \textcircled{-11} & -3 & -1 \\ 0 & -22 & -6 & -5 \end{array} \right) \begin{array}{l} \leftarrow 3 \cdot \\ \leftarrow 3 \cdot \\ \downarrow -2 \end{array} \rightarrow \left(\begin{array}{ccc|c} \textcircled{3} & 4 & 3 & 2 \\ 0 & \textcircled{-11/3} & -1 & -1/3 \\ 0 & -22/3 & -2 & -5/3 \end{array} \right) \begin{array}{l} \downarrow -2 \end{array}$$

$$\downarrow \left(\begin{array}{ccc|c} \textcircled{3} & 4 & 3 & 2 \\ 0 & \textcircled{-11} & -3 & -1 \\ 0 & 0 & 0 & \textcircled{-3} \end{array} \right)$$

trappeterm, ingen løsn.

$$\downarrow \left(\begin{array}{ccc|c} \textcircled{3} & 4 & 3 & 2 \\ 0 & \textcircled{-11/3} & -1 & -1/3 \\ 0 & 0 & 0 & \textcircled{-1} \end{array} \right)$$

trappeterm, ingen løsn.Alt:

$$\left(\begin{array}{ccc|c} \textcircled{3} & 4 & 3 & 2 \\ 2 & -1 & 1 & 1 \\ 7 & 2 & 5 & 3 \end{array} \right) \begin{array}{l} \uparrow -1 \\ \leftarrow -7 \end{array} \rightarrow \left(\begin{array}{ccc|c} \textcircled{1} & 5 & 2 & 1 \\ 2 & -1 & 1 & 1 \\ 7 & 2 & 5 & 3 \end{array} \right) \begin{array}{l} \downarrow -2 \\ \leftarrow -7 \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|c} \textcircled{1} & 5 & 2 & 1 \\ 0 & \textcircled{-4} & -3 & -1 \\ 0 & -33 & -9 & -6 \end{array} \right) \begin{array}{l} \downarrow -3 \end{array} \rightarrow \left(\begin{array}{ccc|c} \textcircled{1} & 5 & 2 & 1 \\ 0 & \textcircled{-11} & -3 & -1 \\ 0 & 0 & 0 & \textcircled{-3} \end{array} \right)$$

trappeterm, ingen løsn

$$7.) \left(\begin{array}{cccc|c} \textcircled{1} & 1 & 1 & 1 & 10 \\ & 2 & 4 & -1 & 7 \\ & -1 & 1 & 1 & 16 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cccc|c} \textcircled{1} & 1 & 1 & 1 & 10 \\ 0 & \textcircled{1} & 3 & -2 & -3 \\ 0 & -2 & 0 & 10 & 6 \end{array} \right) \xrightarrow{R_3 + 2R_2} \left(\begin{array}{cccc|c} \textcircled{1} & 1 & 1 & 1 & 10 \\ 0 & \textcircled{1} & 3 & -2 & -3 \\ 0 & 0 & \textcircled{6} & 6 & 0 \end{array} \right)$$

trappeform

$$\begin{array}{r} x + y + z + w = 10 \\ y + 3z - 2w = -3 \\ 6z + 6w = 0 \end{array}$$

x, y, z : avhengige variable
 w : fri variabel

$$x = -5w + 13$$

$$x + (5w - 3) + (-w) + w = 10$$

$$y + 3(-w) - 2w = -3 \quad y = 5w - 3$$

$$\frac{6z}{6} = -\frac{6w}{6} \quad z = -w$$

Løsning: $(x, y, z, w) = (-5w + 13, 5w - 3, -w, w)$

der w er fri = en parameter

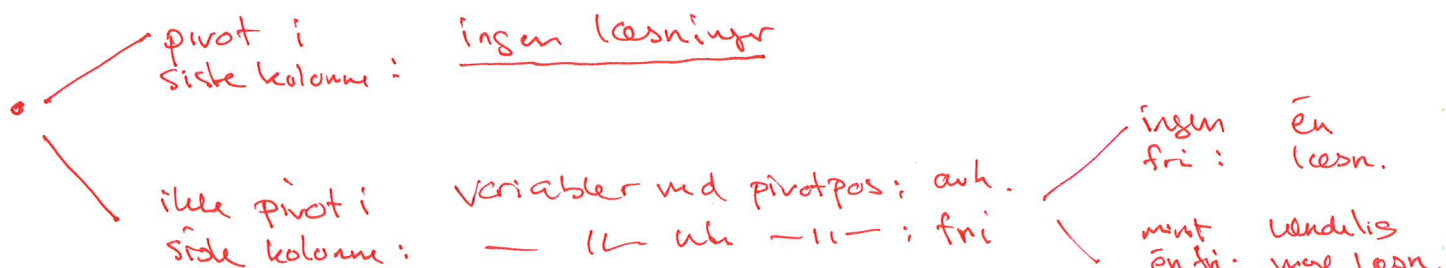
$$= (-5t + 13, 5t - 3, -t, t)$$

uendelig mange løsninger

② Antall løsninger

Musk: - marker pivotposisjonene $\textcircled{}$ og sjekk at du har en trappeform

- antall løsninger:



Teorem:

Enhvert lineært system har enten:

- | | | |
|----------------------------|---|---------------------------|
| - én løsning | } | helt <u>konsistent</u> |
| - uendelig mange løsninger | | |
| - ingen løsning | } | <u>inkonsistent</u> |

Antall løsninger er bestemt av pivotposisjonene.

Ark 8b) $2xy + y^3 + y^2 = 0$
 $x^2 + 3xy^2 + 2xy = 0$

Løsni:

$$(x,y) = \underline{(0,0)}, \underline{(0,-1)},$$

$$\underline{(3/25, -3/5)}$$

$$y(2x + y^2 + y) = 0 \quad y=0 \text{ eller } 2x + y^2 + y = 0$$

$$x(x + 3y^2 + 2y) = 0 \quad x=0 \text{ eller } x + 3y^2 + 2y = 0$$

a) $y=0, x=0$: $(x,y) = \underline{(0,0)}$

b) $y=0, x + 3y^2 + 2y = 0$ $(x,y) = \underline{(0,0)}$

c) $x=0, 2x + y^2 + y = 0$ $y^2 + y = 0$ $y(y+1) = 0$ $y=0$ $\underline{(0,0)}$
 $y=-1$ $\underline{(0,-1)}$

d) $2x - y^2 + y = 0, x + 3y^2 + 2y = 0$
 $x = \underline{-3y^2 - 2y}$

$$2(-3y^2 - 2y) + y^2 + y = 0$$

$$(x,y) = \underline{(0,0)}, \underline{(3/25, -3/5)}$$

$$-5y^2 - 3y = 0$$

$$-y(5y + 3) = 0$$

$$y=0 \text{ eller } y = \underline{-3/5}$$

$$\underline{x=0}$$

$$x = -3\left(\frac{9}{25}\right) - 2\left(-\frac{3}{5}\right) = \frac{-27}{25} + \frac{6 \cdot 5}{5 \cdot 5} = \frac{3}{25}$$

③ Gauss-Jordan eliminering: variant av
Gauss-eliminering

Def: En reduert trappetform

er en trappetform som oppfylter

tilleggskravene:

i) alle pivoter = 1

ii) alle elementer over en pivot = 0

Ek:

$$\begin{aligned} x + y + z + w &= 10 \\ x + 2y + 4z - w &= 7 \\ x - y + z + 4w &= 16 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 1 & 2 & 4 & -1 & 7 \\ 1 & -1 & 1 & 4 & 16 \end{array} \right) \begin{array}{l} \downarrow -1 \\ \downarrow -1 \end{array}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 0 & 1 & 3 & -2 & -3 \\ 0 & -2 & 0 & 3 & 6 \end{array} \right) \begin{array}{l} \downarrow 2 \\ \downarrow -1 \end{array} \rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 0 & 1 & 3 & -2 & -3 \\ 0 & 0 & 6 & 5 & 0 \end{array} \right) \cdot \frac{1}{6}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 10 \\ 0 & 1 & 3 & -2 & -3 \\ 0 & 0 & 1 & 5/6 & 0 \end{array} \right) \begin{array}{l} \downarrow -3 \\ \downarrow -3 \end{array} \rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 10 \\ 0 & 1 & 0 & -5 & -3 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right) \begin{array}{l} \downarrow -1 \\ \downarrow -1 \end{array}$$

trappetform

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 5 & 13 \\ 0 & 1 & 0 & -5 & -3 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right)$$

$$\begin{aligned} x + 5w &= 13 & x &= -5w + 13 \\ y - 5w &= -3 & y &= 5w - 3 \\ z + w &= 0 & z &= -w \end{aligned}$$

reduert
trappetform

Fakta: Den reduserte trappematen til en matrise er entals.