	Emne	Lærebok	Oppgaver
1	Bestemte integral som areal	[E] 5.6	[E] 5.6.1 - 5.6.5
2	Økonomiske anvendelser av integrasjon	[E] 5.7	[E] 5.7.1 - 5.7.6

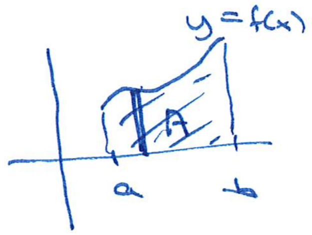
① Bestemt integral som areal

$$\int_a^b f(x)\,dx = \big[F(x)\big]_a^b = F(b)-F(a) \quad \text{når} \quad \begin{cases} F'(x) = f(x) \\ f(x) \text{ kontinuerlig} \\ \quad \text{fn. på } [a,b] \end{cases}$$

Areal:

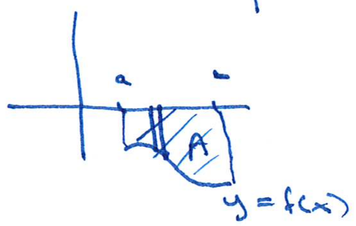
i) $f(x) \geq 0$ i $[a,b]$:

$$A = \int_a^b f(x)\,dx$$



høyde $= f(x)$

ii) $f(x) \leq 0$ i $[a,b]$:

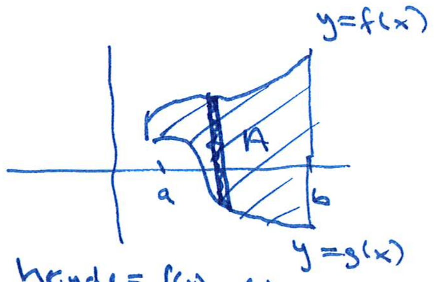


høyde $= -f(x)$

$$-A = \int_a^b f(x)\,dx$$

$$A = \int_a^b -f(x)\,dx$$

iii) $f(x) \geq g(x)$ i $[a,b]$:



høyde $= f(x)-g(x)$

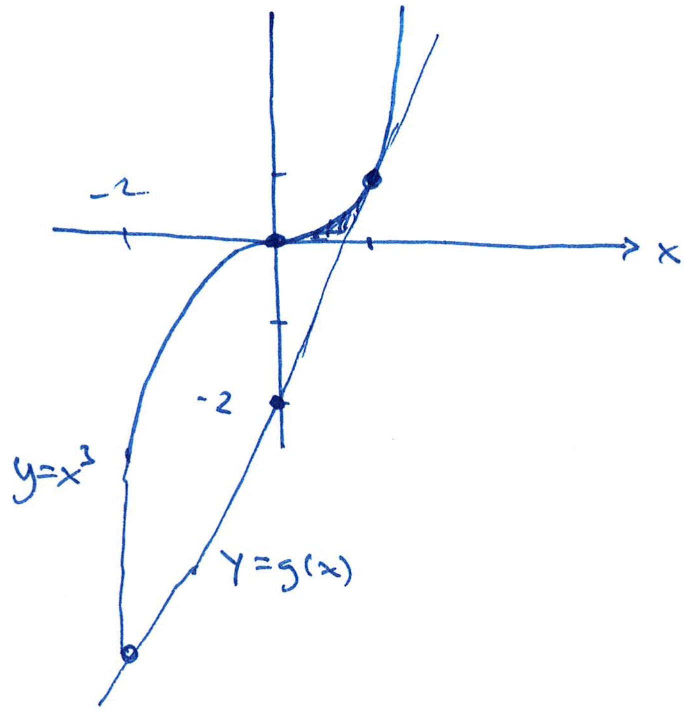
$$A = \int_a^b f(x)-g(x)\,dx$$

Felles for alle tilfellene:

Areal = summen av tynne striper $= \sum_{x=a}^{b} \text{høyde} \cdot \Delta x \rightsquigarrow \int_a^b \begin{Bmatrix} f(x) \\ -f(x) \\ f(x)-g(x) \end{Bmatrix} dx$

Eks: Finn arealet av område begrenset av

$f(x) = x^3$, $g(x) = 3x-2$ og x-aksen.



$$x^3 = 3x-2$$

$$\underline{x^3 - 3x + 2 = 0}$$

$x=1$: $1^3 - 3\cdot 1 + 2 = 0$ $\underline{\text{ok}}$

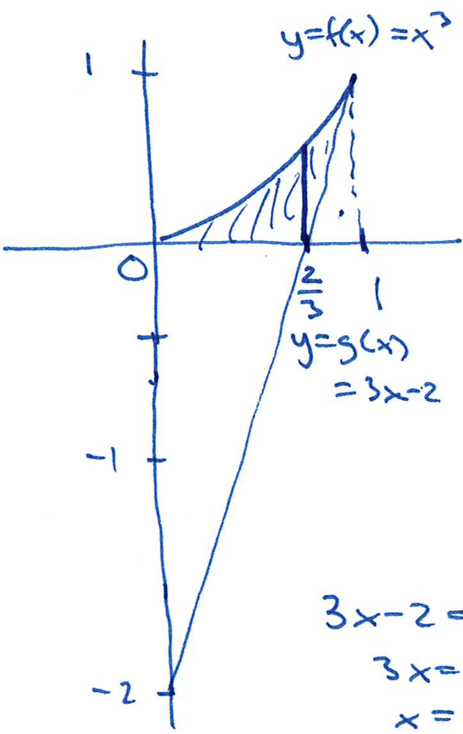
$$x^3 - 3x + 2 : (x-1) = x^2 + x - 2$$
$$\underline{-\; x^3 - x^2}$$
$$x^2 - 3x + 2$$
$$\underline{-\; x^2 - x}$$
$$-2x + 2$$
$$\underline{-2x + 2}$$
$$0$$

$$(x-1)\cdot(x^2 + x - 2) = 0$$

$\underline{x=1}$, $x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\underline{x=-2}$ $\underline{x=1}$



$3x - 2 = 0$

$3x = 2$

$x = 2/3$

$\underline{\text{Alt I:}}$ $\displaystyle\int_{2/3}^{1} 3x - 2\,dx$ ''

$$\int_0^1 x^3\,dx - \Delta$$

$$= \left[\tfrac{1}{4}x^4\right]_0^1 - \left[\frac{3x^2}{2} - 2x\right]_{2/3}^{1}$$

$$= \left(\tfrac{1}{4} - 0\right) - \left[\left(3/2 - 2\right) - \left(\tfrac{3}{2}\cdot\left(\tfrac{2}{3}\right)^2 - 2\cdot\tfrac{2}{3}\right)\right]$$

$$= 1/4 - (-1/2) + \left(\tfrac{2}{3} - \tfrac{4}{3}\right)$$

$$= 1/4 + \underbrace{1/2 - 2/3}_{-1/6} = \frac{3}{12} + \frac{6}{12} - \frac{8}{12} = \underline{\underline{\frac{1}{12}}}$$

Alt 2:

$$A = \int_0^{2/3} x^3\,dx + \int_{2/3}^{1} x^3 - (3x-2)\,dx$$

$$= \int_0^{2/3} x^3\,dx + \int_{2/3}^{1} x^3 - 3x + 2\,dx$$

$$= \underbrace{\int_0^{2/3} x^3\,dx + \int_{2/3}^{1} x^3\,dx}_{\int_0^1 x^3\,dx} + \int_{2/3}^{1} -3x+2\,dx$$

Nyttig formel:

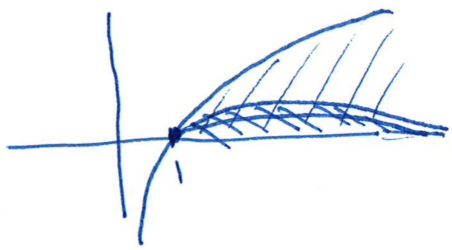
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Uegentlig integral:

Eks:

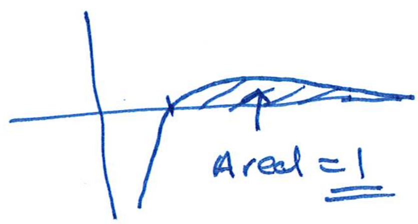
$$\int_1^{\infty} \ln x\,dx = \left[x\ln x - x\right]_1^{\infty} = \lim_{x\to\infty}(x\ln x - x) - (1\cdot \underset{0}{\ln 1} - 1)$$

$$= \lim_{x\to\infty} \underset{\to\infty}{x}\,(\underset{\to\infty}{\ln x - 1}) + 1 = \underline{\underline{\infty}}$$



$$\int_1^{\infty} \frac{\ln x}{x}\,dx = \left[\tfrac{1}{2}(\ln x)^2\right]_1^{\infty} = \underline{\underline{\infty}}$$

$$\int_1^{\infty} \frac{\ln x}{x^2}\,dx = \left[-\tfrac{1}{x}\ln x - \tfrac{1}{x}\right]_1^{\infty}$$



Areal $= \underline{\underline{1}}$

$$= \underbrace{\lim_{x\to\infty} \frac{-\ln x - 1}{x}}_{=0} - (-1) = \underline{\underline{1}}$$

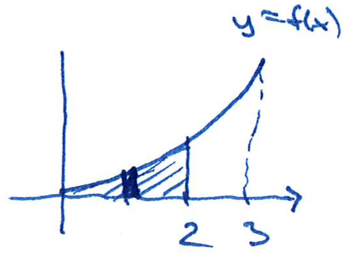
② Anvendelser av integrasjon

Integral = summen av "noe" som endrer seg kontinuerlig

i) Sannsynlighetsregning

X : kontinuerlig stokastisk variabel

Eks: X kontinuerlig stokastisk variabel med utfallsrom $[0,3]$ og tetthetsfunksjon $f(x) = ax^2$



$$p(0 \leq x \leq 3) = \int_0^3 f(x)\,dx = \left[a \cdot \tfrac{1}{3}x^3\right]_0^3 = a \cdot \tfrac{1}{\cancel{3}} \cdot 3^{\cancel{3}\,2} - 0 = 9a = 1$$

$a = 1/9$

$$\boxed{f(x) = \frac{1}{9}x^2, \quad 0 \leq x \leq 3}$$

$$p(x \leq 2) = \int_0^2 f(x)\,dx = \int_0^2 \frac{1}{9}x^2\,dx = \left[\frac{1}{9} \cdot \frac{1}{3}x^3\right]_0^2$$

$$= \frac{1}{27} \cdot 2^3 - 0 = 8/27$$

Husk: (*) En tetthetsfn. $f(x)$ til en kont. stokastisk variabel oppfyller

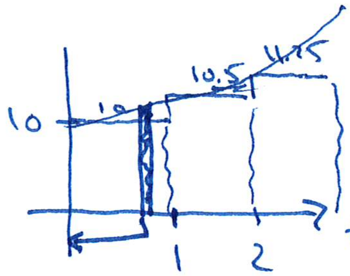
i) $f(x) \geq 0$ for alle x

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(*) $p(c \leq x \leq d) = \int_c^d f(x)\,dx$

ii) Kontinuerlige kontantstrømmer

Eks: Leieinntekt 10 mNOK per år, som øker med 5% i året



Total kontantstrøm over 3 år:

10 + 10.5 + 11.25

= 31.75 mNOK

Kont. kontantstrøm:

$$f(t) = 10 \cdot 1.05^t \quad \text{mNOK/år}$$

Total kontantstrøm de første 3 årene:

$$\int_0^3 f(t)\,dt = \int_0^3 10 \cdot \underline{1.05^t}\,dt$$

$$= \left[10 \cdot \frac{1}{\ln(1.05)}\, 1.05^t \right]_0^3$$

$$= \frac{10}{\ln(1.05)} \cdot \left(1.05^3 - 1 \right) \simeq \underline{\underline{32.3}} \text{ mNOK}$$

$$\int a^x\,dx = \frac{1}{\ln a} \cdot a^x + C$$

eller

$$a^x = e^{\ln(a^x)} = e^{x \cdot \ln a}$$

Nåverdi: Disk. rate = 10%

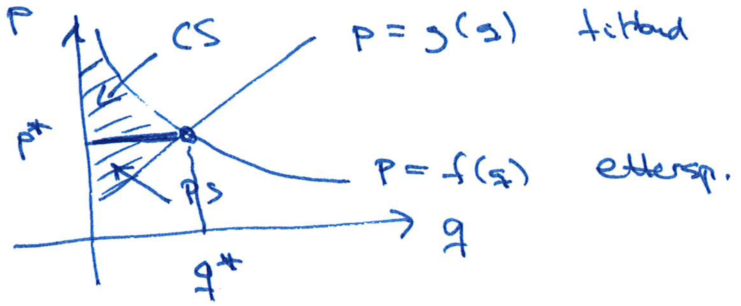
$$\int_0^3 f(t) \cdot e^{-0.10t}\,dt = \int_0^3 10 \cdot 1.05^t \cdot e^{-0.10t}\,dt$$

$$= \int_0^3 10 \cdot \underbrace{e^{\ln(1.05)\cdot t} \cdot e^{-0.10t}}_{e^{\ln(1.05)t - 0.10t}}\,dt = \ldots$$

Substitusjon

unødvendig

iii) Konsument/produsentoverskudd:



CS = konsumentoverskudd

PS = produsentoverskudd

$$CS = \int_0^{q^*} f(q) - p^* \, dq \qquad PS = \int_0^{q^*} p^* - g(q) \, dq$$

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