	Emne	Lærebok	Oppgaver
1	Repetisjon		
2	Oppgavegjennomgang		[Ark] 3j, 6b, 8b, 9
3	Arealberegning og bestemte integral	[E] 5.6	[E] 5.6.3 - 5.6.5

① Repetisjon: integral rasj. fu.; bestemte integral

② Oppgaveark 28

3j $\int_{-1}^{1} e^x + e^{-x}\,dx = \left[e^x - e^{-x}\right]_{-1}^{1}$

$= (e^1 - e^{-1}) - (e^{-1} - e^1) = \underline{\underline{2e - \frac{2}{e}}} = 2(e - 1/e)$

6b $\int x^3\sqrt{x^2+4}\,dx = \int x^{\cancel{3}\,2} \cdot \sqrt{u} \cdot \frac{1}{2\cancel{x}}\,du$

$\boxed{\begin{aligned} u &= x^2+4 \\ du &= 2x\,dx \end{aligned}}$ $\quad x^2 = u-4 \quad dx = \frac{1}{2x}\,du$

$= \frac{1}{2}\int x^2\sqrt{u}\,du = \frac{1}{2}\int (u-4)u^{1/2}\,du$

$= \frac{1}{2}\int u^{3/2} - 4u^{1/2}\,du = \frac{1}{2}\left(\frac{2}{5}u^{5/2} - 4\cdot\frac{2}{3}u^{3/2}\right) + C$

$= \underline{\underline{\frac{1}{5}(x^2+4)^{5/2} - \frac{4}{3}(x^2+4)^{3/2} + C}}$

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8b) $\int_{0=x}^{1=x} 15x\sqrt{x+1}\,dx = \int 15x\sqrt{u}\,du = \int_1^2 15(u-1)\sqrt{u}\,du$

$x=1: \quad u=2$
$x=0: \quad u=1$

$\boxed{\begin{array}{l} u=x+1 \\ du=dx \end{array}}$ $\quad x=u-1$

$= 15\int_1^2 u^{3/2}-u^{1/2}\,du = 15\left[\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right]_1^2$

$= \left[6u^{5/2}-10u^{3/2}\right]_1^2 = \left(6\cdot 2^2\cdot\sqrt{2}-10\cdot 2\cdot\sqrt{2}\right)-(6-10)$

$= 24\sqrt{2}-20\sqrt{2}+4 = \underline{\underline{4\sqrt{2}+4}}$

9.) $\int_{-1}^{1} \frac{e^x}{e^x+1}\,dx = \left[\ln(e^x+1)\right]_{-1}^{1} = \ln(e+1)-\ln(e^{-1}+1)$

$\left(\int \frac{e^x}{e^x+1}\,dx = \int \frac{\cancel{e^x}}{u}\cdot\frac{1}{\cancel{e^x}}\,du = \int \frac{1}{u}\,du \right.$

$\boxed{\begin{array}{l} u=e^x+1 \\ du=e^x\cdot dx \end{array}}$

$\left. = \ln|u|+C = \underline{\ln(e^x+1)+C} \right)$

$\ln a - \ln b = \ln(a/b)$

$= \ln\frac{e+1}{e^{-1}+1} = \ln\frac{e+1}{1/e+1}\frac{\cdot e}{\cdot e}$

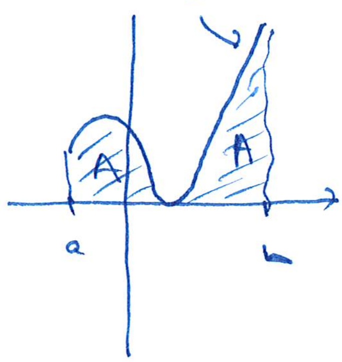
$= \ln\frac{e\cancel{(e+1)}}{\cancel{1+e}} = \ln e = \underline{\underline{1}}$

③ Arealberegning og bestemte integral

Fundamentalsetningen for integralregning

La $f(x)$ være en kontinuerlig funksjon på intervallet $[a,b]$, og $f(x) \geq 0$ for alle x i $[a,b]$. Da er arealet under grafen til f (dvs mellom x-aksen og grafen til f) i intervallet $[a,b]$ gitt ved

$$A = \int_a^b f(x)\,dx$$



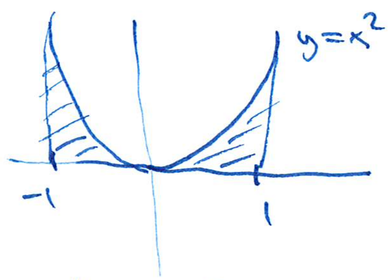
$$A = \int_a^b f(x)\,dx$$

Eks:

$$\int_{-1}^{1} x^2\,dx = \left[\tfrac{1}{3}x^3\right]_{-1}^{1}$$

$$= \tfrac{1}{3}\cdot 1^3 - \tfrac{1}{3}(-1)^3 =$$

$$= \tfrac{1}{3} + \tfrac{1}{3} = \underline{\underline{2/3}}$$



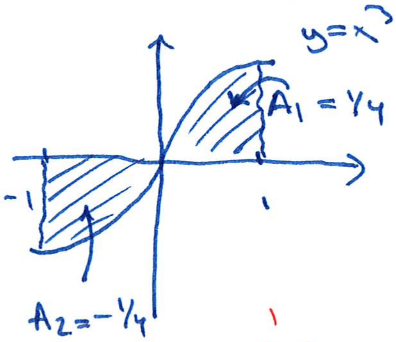
Areal = $\underline{\underline{2/3}}$

Eks:

$$\int_{-1}^{1} x^3\,dx = \left[\tfrac{1}{4}x^4\right]_{-1}^{1}$$

$$= \tfrac{1}{4}\cdot 1^4 - \tfrac{1}{4}(-1)^4$$

$$= 1/4 - 1/4 = \underline{\underline{0}}$$



$$\int_0^1 x^3\,dx = \left[\tfrac{1}{4}x^4\right]_0^1 = 1/4 - 0 = \underline{1/4} = A_1$$

$$\int_{-1}^{0} x^3\,dx = \left[\tfrac{1}{4}x^4\right]_{-1}^{0} = 0 - (+1/4) = \underline{-1/4} = -A_2$$

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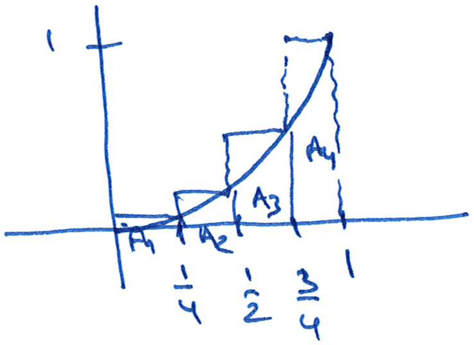
Eks: Arealet mellom x-aksen og $f(x)=x^2$ på $[0,1]$



n = 4:

$A_4 \simeq \frac{1}{4}\cdot 1^2 = 1/4$

$A_3 \simeq \frac{1}{4}\cdot (3/4)^2 = 3/64$

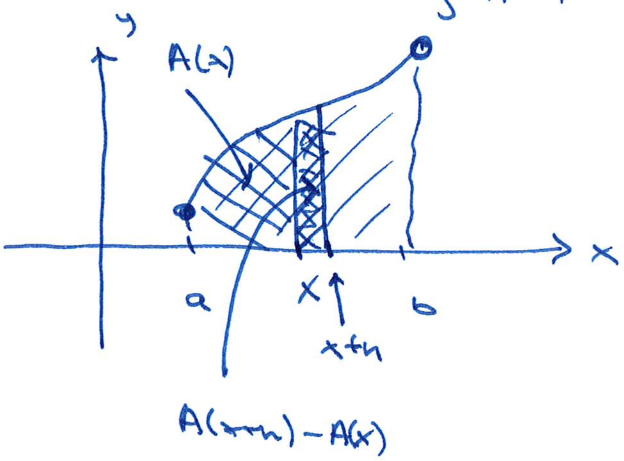
$A_2 \simeq$

$A_1 \simeq$

$A = A_1 + A_2 + A_3 + A_4$

$n \to \infty$

Forklaring: "bevis"



$A =$ arealet under grafen til f

$A(x) =$ arealet under grafen til f i intervallet $[a,x]$

Da har vi: $A(b) = A$

$A(a) = 0$

$$A'(x) = \lim_{h\to 0} \frac{A(x+h)-A(x)}{h}$$

h liten $\simeq \dfrac{A(x+h)-A(x)}{h}$

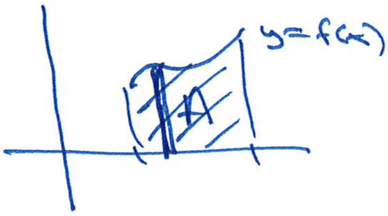
$$= \frac{\cancel{h}\cdot f(x)}{\cancel{h}} = f(x)$$

$$\int_a^b f(x)\,dx = \Big[A(x)\Big]_a^b$$

$$= A(b) - A(a) = A - 0 = \underline{\underline{A}}$$

i) Tilfelle I:

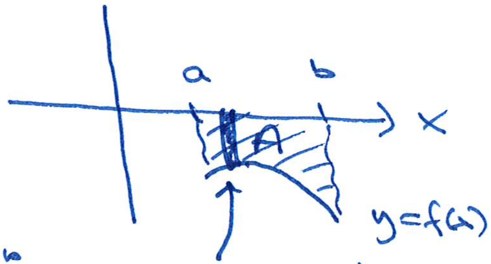
$f(x) \geq 0$ på $[a,b]$



$$\boxed{A = \int_a^b f(x)\,dx}$$

ii) Tilfelle 2:

$f(x) \leq 0$ på $[a,b]$



$$\int_a^b (0 - f(x))\,dx = \int_a^b -f(x)\,dx$$

$$\int_a^b f(x) = -A$$

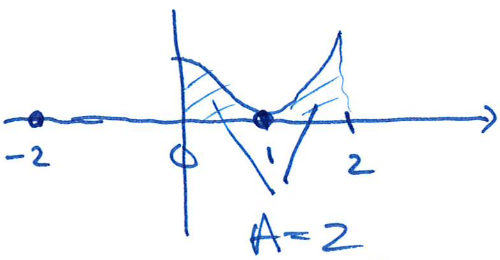
$$A = -\int_a^b f(x)\,dx = \int_a^b -f(x)\,dx$$

$$\boxed{A = \int_a^b -f(x)\,dx}$$

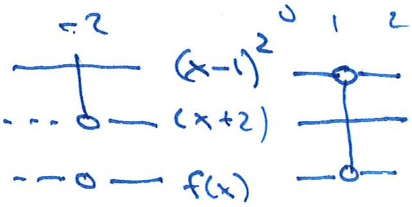
Eks: $\displaystyle\int_0^2 x^3 - 3x + 2\,dx = \left[\frac{1}{4}x^4 - 3\cdot\frac{1}{2}x^2 + 2x\right]_0^2$

$$= \left(\frac{1}{4}\cdot 2^4 - \frac{3}{2}\cdot 2^2 + 2\cdot 2\right) - (0) = 4 - 6 + 4 = \underline{\underline{2}}$$

$$= \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx$$



$A = \underline{\underline{2}}$



$f(x) = 0$

$x^3 - 3x + 2 = 0$

$(x-1)(x^2 + x - 2) = 0$

$\boxed{(x-1)(x+2)(x-1) = 0}$

$x^3 - 3x + 2 : x - 1 = x^2 + x - 2$

$x^3 - x^2$

$x^2 - 3x + 2$

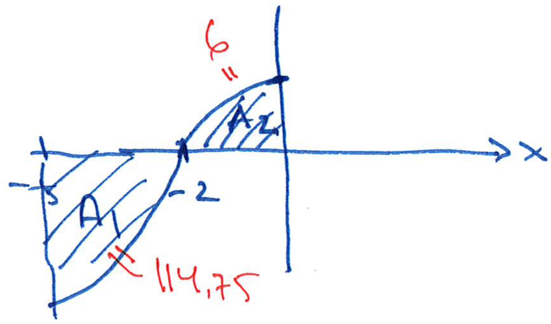
$x^2 - x$

$-2x + 2$

$-2x + 2$

0

$$\int_{-5}^{0} x^3-3x+2\,dx =$$



$$= \int_{-5}^{-2} f(x)\,dx + \int_{-2}^{0} f(x)\,dx$$

$$= -A_1 + A_2$$

$$\int_{-5}^{0} x^3-3x+2\,dx = \left[\frac{1}{4}x^4-\frac{3}{2}x^2+2x\right]_{-5}^{0} = \boxed{A_2-A_1}$$

$$=(0)-\left(\frac{5^4}{4}-\frac{3}{2}\cdot 25-10\right) = -\frac{625}{4}+\frac{75}{2}+10$$

$$= 47.5 - 156{,}25 = \underline{\underline{-108{,}75}}$$

$$A_1+A_2 = \int_{-5}^{-2} -f(x)\,dx + \int_{-2}^{0} f(x)\,dx$$

$$-\left[\frac{1}{4}x^4-\frac{3}{2}x^2+2x\right]_{-5}^{-2}$$

$$= -\left[\left(\frac{2^4}{4}-\frac{3\cdot 2^2}{2}-4\right)-\left(\frac{5^4}{4}-\frac{3\cdot 5^2}{2}-10\right)\right]$$

$$= -(4-6-4)+\left(\frac{625}{4}-\frac{75}{2}-10\right)$$

$$= 6+108.75 = \underline{\underline{114.75}}$$

$$\left[\frac{1}{4}x^4-\frac{3}{2}x^2+2x\right]_{-2}^{0}$$

$$= (0)-(4-6-4)$$

$$= \underline{\underline{6}}$$

Sjekk: $-A_1+A_2 = -114.75+6$

$$= -108.75$$

$$= \int_{-5}^{0} x^3-3x+2\,dx \quad ✓ \quad \underline{\underline{ok}}$$