	Emne	Lærebok	Oppgaver
1	Delbrøkoppspaltning	[E] 5.5	5.5.1 - 5.5.6
2	Bestemte integral	[E] 5.6	5.6.1 - 5.6.2

Regne ut ubestemte integral:

– integrasjonsregler

$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \qquad n \neq -1$

$\int \frac{1}{x} \, dx = \ln|x| + C$

$\int e^x \, dx = e^x + C$

– integrasjonsteknikker

delvis int.: $\int u'v \, dx = uv - \int uv' \, dx$

substitusjon: $\int f(x) \, dx$ $\boxed{u = \ldots \quad du = u' \cdot dx}$

integrasjon av rasjonale funksjoner:

i) polynomdivisjon: grad til teller ≥ graden til nevner

ii) nevner har grad én:

$$\boxed{\int \frac{A}{ax+b} \, dx = \frac{A}{a} \ln|ax+b| + C}$$

Delbrøksoppspaltning:

Eks: $\int \frac{5x-2}{x^2-x} \, dx$

i) Faktoriser nevner:

$x^2 - x = x(x-1)$

ii) Skriv ned:

$$\frac{5x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

~~Substitusjon?~~ Nei

~~$u = x^2 - x$~~

~~$du = (2x-1) \cdot dx$~~

~~$\int \frac{5x-2}{u} \frac{du}{2x-1}$~~

~~$= \int \frac{5x-2}{2x-1} \frac{1}{u} \, du$~~

$\int \frac{5x-2}{u} \frac{1}{2x-1} \, du$

iii) Finn konstantene A og B:

$$\frac{5x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \quad \Big| \cdot x(x-1)$$

$$5x-2 = A(x-1) + Bx$$

$$= \underline{Ax} - A + \underline{Bx}$$

$$\underline{5x}\ \underline{-2} = \underbrace{(A+B)}_{=5}x + \underbrace{(-A)}_{=-2}$$

$$A+B = 5 \qquad \underline{B=3}$$

$$-A = -2 \qquad \underline{A=2}$$

Alt: $5x-2 = A(x-1)+Bx$

$x=1$: $3 = B \to B=3$

$x=0$: $-2 = A\cdot(-1) \to A=2$

Feil: $\cancel{5x - Ax - Bx = 2 - A}$

$$\cancel{x = \frac{2-A}{5-A-B}}$$

iv) Løs integralet

$$\frac{5x-2}{x(x-1)} = \frac{2}{x} + \frac{3}{x-1} \qquad \text{delbrøksoppspaltning}$$

$$\int \frac{5x-2}{x(x-1)}\,dx = \int \frac{2}{x} + \frac{3}{x-1}\,dx = \int \frac{2}{x}\,dx + \int \frac{3}{x-1}\,dx$$

$$= \underline{\underline{2\ln|x| + 3\ln|x-1| + C}}$$

Eks: $\displaystyle\int \frac{x^2}{x^2-4}\,dx$

$$x^2-4 = (x-2)(x+2)$$

$$\frac{x^2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} \quad \Big|\cdot (x-2)(x+2)$$

$$x^2 = A(x+2) + B(x-2)$$

$$\cancel{x^2 = A(x+2) + B(x-2)}$$

$$\cancel{x=-2: \quad 4 = B\cdot(-4) \qquad B=-1}$$

$$\cancel{x=2: \quad 4 = 4A \qquad A=1}$$

Alt:

$$x^2 = Ax + 2A + Bx - 2B$$

$$x^2 = (A+B)x + (2A-2B)$$

umulig

Polynomdivisjon:

$$\begin{array}{l} x^2 : x^2-4 = 1 \\ -(x^2-4) \\ \hline \quad 4 \end{array}$$

$$\frac{x^2}{x^2-4} = 1 + \frac{4}{x^2-4}$$

$$\int \frac{x^2}{x^2-4}\,dx = \int 1 + \frac{4}{x^2-4}\,dx$$

$$= x + \int \frac{4}{x^2-4}\,dx$$

$$= x + \int \frac{1}{x-2}\,dx + \int \frac{-1}{x+2}\,dx$$

$$= \underline{\underline{x + \ln|x-2| - \ln|x+2| + C}}$$

Delbrøk:

$$\frac{4}{(x-2)(x+2)} = \frac{\overset{=1}{A}}{x-2} + \frac{\overset{=-1}{B}}{x+2} \quad \Big|\cdot (x-2)(x+2)$$

$$4 = A(x+2) + B(x-2)$$

$$x=-2: \quad 4 = -4B \qquad B=-1$$

$$x=2: \quad 4 = 4A \qquad A=1$$

Eks: $\displaystyle\int \frac{x}{x^2-4x+4}\,dx = \int \frac{1}{x-2} + \frac{2}{(x-2)^2}\,dx$

$\displaystyle = \ln|x-2| - \frac{2}{x-2} + C$

Delbrøk:

$x^2-4x+4 = (x-2)^2 = (x-2)\cdot(x-2)$

$\displaystyle \frac{x}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{x-2} = \frac{A+B}{x-2} \quad \Big| \cdot (x-2)^2$

$x = (A+B)(x-2) = (A+B)x + (-2(A+B))$

Umulig

$\displaystyle \frac{x}{(x-2)^2} = \frac{A^{=1}}{x-2} + \frac{B^{=2}}{(x-2)^2} \quad \Big| \cdot (x-2)^2$

$x = A(x-2) + B$

$x=2$: $\quad 2 = B \qquad B = 2$

$x=0$: $\quad 0 = -2A + B \qquad 2A = B = 2$

$A = 1$

$\displaystyle\int \frac{2}{(x-2)^2}\,dx$

$\boxed{\begin{array}{l} u = x-2 \\ du = 1\cdot dx \end{array}}$

$\displaystyle = \int \frac{2}{u^2}\,du = \int 2u^{-2}\,du$

$\displaystyle = 2\cdot\frac{u^{-1}}{-1} + C$

$\displaystyle = -2\cdot\frac{1}{x-2} + C$

Eks: $\displaystyle\int \frac{1}{x^2+1}\,dx = \arctan(x) + C$

x^2+1 kan ikke faktoriseres i lineære faktorer

$\Rightarrow$ delbrøk fungerer ikke

Eks: $\int \frac{x^2}{x^3-x}\,dx = \int \frac{1/2}{x-1}\,dx + \int \frac{1/2}{x+1}\,dx$

$= \underline{\underline{\frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| + C}}$

$x^3 - x = x(x^2-1) = \underline{x(x-1)(x+1)}$

$\frac{x^2}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \quad \Big| \cdot x(x-1)(x+1)$ ($A=0$, $B=1/2$, $C=1/2$)

$x^2 = A(x^2-1) + B(x^2+x) + C(x^2-x)$

$x=0: \quad 0 = -A \qquad \underline{A=0}$

$x=1: \quad 1 = 2B \qquad \underline{B=1/2}$

$x=-1: \quad 1 = 2C \qquad \underline{C=1/2}$

2 **Bestemte integral:** Anta $f(x)$ er kont. på $[a,b]$

Hvis $\int f(x)\,dx = F(x)+C$, så er $\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b)-F(a)$

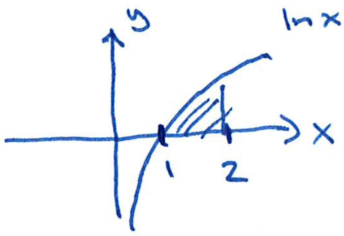
Eks: $\int_1^2 \ln x\,dx = \Big[x\cdot\ln x - x\Big]_1^2 = (2\ln 2 - 2) - (1\cdot\ln 1 - 1)$

$= \underline{2\ln 2 - 1} \simeq \underline{0.38}$

$\int_1^2 1\cdot\ln x\,dx = \Big[x\cdot\ln x\Big]_1^2 - \int_1^2 \underbrace{x\cdot\frac{1}{x}}_{1}\,dx = \Big[x\cdot\ln x - x\Big]_1^2$

$\boxed{\begin{array}{ll} u = x & v = \ln x \\ u' = 1 & v' = 1/x \end{array}}$

$\int 1\cdot\ln x\,dx = x\cdot\ln x - \int x\cdot\frac{1}{x}\,dx = \underline{x\ln x - x} + C$



Eks:

$$\int_0^5 \frac{7}{x^2-5x-6}\,dx = \Big[\ln|x-6| - \ln|x+1|\Big]_0^5$$

$$= \big(\ln|-1| - \ln|6|\big) - \big(\ln|-6| - \ln|1|\big)$$

$$= (0 - \ln 6) - (\ln 6 - 0)$$

$$= \underline{\underline{-2\ln 6}}$$

$$x^2-5x-6 = (x-6)(x+1)$$

$$\frac{7}{(x-6)(x+1)} = \frac{A^{=1}}{x-6} + \frac{B^{=-1}}{x+1}$$

$$7 = A(x+1) + B(x-6)$$

$x=-1$: $7 = -7B$ $\quad B = -1$

$x = 6$: $7 = 7A$ $\quad A = 1$

Merk: $f(x) = \dfrac{7}{x^2-5x-6}$ kont i $[0,5]$