	Emne	Lærebok	Oppgaver
1	Substitusjon	[E] 5.3	5.3.1 - 5.3.3
2	Delvis integrasjon	[E] 5.4	5.4.1 - 5.4.5

Repetisjon: $\int f(x)\,dx = F(x) + C$

Int. regler: $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \quad (n \neq -1)$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

② Delvis integrasjon

$$\boxed{\int u'v\,dx = uv - \int uv'\,dx}$$

Formel for delvis integrasjon

produktregel for derivasjon

$$(uv)' = u'v + uv'$$

$$\int (uv)'\,dx = \int u'v\,dx + \int uv'\,dx$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

Eks: $\int x \cdot e^x\,dx$

$= \int u'v\,dx$ $\quad \boxed{\begin{array}{ll} u = \frac{1}{2}x^2 & v = e^x \\ u' = x & v' = e^x \end{array}}$

$= \frac{1}{2}x^2 e^x - \int \frac{1}{2}x^2 \cdot e^x\,dx$

$$\int x \cdot e^x\,dx = \int u'v\,dx = xe^x - \int e^x \cdot 1\,dx$$

$\boxed{\begin{array}{ll} u = e^x & v = x \\ u' = e^x & v' = 1 \end{array}}$

$= xe^x - \int e^x\,dx$

$= xe^x - e^x + C$

1 Substitusjon

Eks:

$$\int e^{2x}\,dx = \int e^u \frac{du}{2} = \int e^u \cdot \tfrac{1}{2}\,du$$

$$\boxed{\begin{aligned} u &= 2x \\ du &= u' \cdot dx = 2dx \end{aligned}}$$

$$\frac{du}{2} = \frac{2dx}{2} \qquad dx = \frac{du}{2}$$

Kjerneregelen: for derivasjon

$$f(x) = g(u(x))$$

$$\Rightarrow f'(x) = g'(u(x)) \cdot u'(x)$$

Eks: $f(x) = e^{2x} = e^u$, $u = 2x$

$$f'(x) = (e^u)' \cdot u' = e^u \cdot 2 = 2e^{2x}$$

$$= \int \tfrac{1}{2} e^u\,du = \tfrac{1}{2}\int e^u\,du = \tfrac{1}{2}(e^u) + C = \frac{1}{2}e^{2x} + C$$

Husk:
- velger et variabelskifte $u = \ldots$ (uttrykk i x)
- skriv om integralet til et integral i u vha $\boxed{\begin{aligned} u &= \ldots \\ du &= u' \cdot dx \end{aligned}}$

  $$\int \underbrace{\ldots}_{\text{(uttrykk i } u)} du$$
- løs integralet i u (og uttrykk svaret ved x)

Eks:

$$\int \frac{1}{1-x}\,dx = \int \frac{1}{u}\frac{du}{-1} = \int -\frac{1}{u}\,du = -\int \frac{1}{u}\,du$$

$$\boxed{\begin{aligned} u &= 1-x \\ du &= -1 \cdot dx \end{aligned}}$$

$$dx = \frac{du}{-1}$$

$$= -\ln|u| + C$$

$$= -\ln|1-x| + C$$

Eks: $\int x \cdot \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}\cancel{x^2} \cdot \frac{1}{\cancel{x}} \, dx$

$$\boxed{\begin{array}{ll} u = \frac{1}{2}x^2 & v = \ln x \\ u' = x & v' = 1/x \end{array}}$$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\left(\frac{1}{2}x^2\right) + C$

$= \underline{\underline{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}}$

Eks: $\int \ln(x) \, dx = \int 1 \cdot \ln(x) \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx$

$$\boxed{\begin{array}{ll} u = x & v = \ln x \\ u' = 1 & v' = 1/x \end{array}}$$

$= x \ln x - \int 1 \, dx = \underline{\underline{x \ln x - x + C}}$

Eks: $\int \frac{\ln x}{x} \, dx = \int \frac{1}{x} \cdot \ln x \, dx = F(x) + C_1$

$$\boxed{\begin{array}{ll} u = \ln x & v = \ln x \\ u' = 1/x & v' = 1/x \end{array}}$$

$= (\ln x)^2 - \int \ln x \cdot \frac{1}{x} \, dx = (\ln x)^2 - (F(x) + C)$

$F(x) + C = (\ln(x))^2 - F(x) - C_2$

$\frac{2F(x)}{2} = \frac{(\ln x)^2 - 2C}{2}$

$F(x) = \frac{(\ln x)^2}{2} - C$

Konkl: $\int \frac{\ln x}{x} \, dx = F(x) + C = \left(\frac{(\ln x)^2}{2} - C_2\right) + C_1 = \underline{\underline{\frac{(\ln x)^2}{2} + C_3}}$

<u>Eks:</u> $\int \frac{x}{1-x^2}\,dx = \int \frac{\cancel{x}}{u}\,\frac{1}{(-2\cancel{x})}\,du = \int -\frac{1}{2}\,\frac{1}{u}\,du$

$$\boxed{\begin{aligned} u &= 1-x^2 \\ du &= -2x\,dx \end{aligned}}$$

$dx = \frac{du}{-2x} = \frac{1}{-2x}\cdot du$

$= -\frac{1}{2}\ln|u| + C$

$= \underline{\underline{-\frac{1}{2}\ln|1-x^2| + C}}$

$\int \frac{\cancel{x}}{1+x^2}\,dx = \int \frac{1}{u}\,\frac{du}{2x} = \int \frac{1}{u}\cdot\frac{1}{2x}\,du$

$$\boxed{\begin{aligned} u &= 1+x^2 \\ du &= 2x\,dx \end{aligned}}$$

$x^2 = u-1$

$x = \pm\sqrt{u-1}$

$= \int \frac{1}{u}\,\frac{1}{2(\pm\sqrt{u-1})}\,du$

<u>Eks:</u> $\int \frac{\ln x}{x}\,dx = \int \frac{1}{x}\cdot\ln x\,dx = \int \frac{1}{\cancel{x}}\cdot u\cdot\cancel{x}\cdot du$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

$\frac{1\cdot x}{1/x\cdot x} = \frac{x}{1}$

$dx = x\cdot du$

$= \int u\,du = \frac{1}{2}u^2 + C = \underline{\underline{\frac{1}{2}(\ln x)^2 + C}}$

Eks:

$$\int \frac{e^{1-\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^{1-u}}{u}\,2\sqrt{x}\,du$$

$$\boxed{\begin{aligned} u &= \sqrt{x} = x^{1/2} \\ du &= \frac{1}{2\sqrt{x}}\,dx \end{aligned}}$$

$$(x^{1/2})' = \frac{1}{2}\cdot x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$dx = 2\sqrt{x}\,du$$

$$= \int \frac{e^{1-u}}{\cancel{u}} \cdot 2\cancel{u}\,du = 2\int e^{1-u}\,du = 2\int e^{v}\,\frac{dv}{-1}$$

$$\boxed{\begin{aligned} v &= 1-u \\ dv &= -1\,du \end{aligned}}$$

$$= \frac{2}{-1}\int e^{v}\,dv = -2e^{v} + C = -2e^{1-u} + C$$

$$= \underline{\underline{-2e^{1-\sqrt{x}} + C}}$$