Mock Exam	Mock Midterm exam in GRA 6035 Mathematics
Date	Fri October 4th, 2024 at 1000 - 1100

This exam consists of 8 problems with score 0 - 3p each, and maximal score on this exam is 24p.
You must give reasons for your answers.

Question 1.
Determine the rank of the matrix A:

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 5 & 2 \\ 4 & 6 & 10 & 6 \end{pmatrix}$$

Question 2.
Find a base of $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 3 \\ 5 \\ 10 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$$

Question 3.
Determine all values of t such that $\mathbf{v}_1$ is in $\text{span}(\mathbf{v}_2, \mathbf{v}_3)$:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ t \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} t \\ 8 \\ -2 \end{pmatrix}$$

Question 4.
Determine the equilibrium state of the Markov chain with transition matrix A, if it exists:

$$A = \begin{pmatrix} 0.52 & 0.16 \\ 0.48 & 0.84 \end{pmatrix}$$

Question 5.
Determine all values of s such that A is diagonalizable:

$$A = \begin{pmatrix} 1 & s & 1 \\ 0 & 1 & s \\ 0 & 0 & 2 \end{pmatrix}$$

Question 6.
Determine the definiteness of the quadratic form $f(x, y, z) = x^2 + 4xy + 6xz + 3y^2 - 10yz + 8z^2$.

Question 7.
Find the range of $f(\mathbf{x}) = \mathbf{x}^T A\mathbf{x} + B\mathbf{x}$ when

$$A = \begin{pmatrix} 3 & 2 & -2 \\ 2 & 4 & 0 \\ -2 & 0 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} -6 & 4 & -2 \end{pmatrix}$$

Question 8.
Show that $\mathbf{v}$ is an eigenvector of A, and use this to find all the eigenvalues of A:

$$A = \begin{pmatrix} 3 & 2 & -2 \\ 2 & 4 & 0 \\ -2 & 0 & 7 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$