BI NORWEGIAN BUSINESS SCHOOL

Written examination:	**GRA 60353**	**Mathematics**	
Examination date:	Dec 2013	3h	Total no. of pages: 2
Permitted examination	A bilingual dictionary and BI-approved calculator TEXAS		
support material:	INSTRUMENTS BA II Plus		
Answer sheets:	Squares		
	Counts 80% of GRA 6035		The subquestions have equal weight
Mock exam			Responsible department: Economics

All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 4(d) can be skipped, but give 6p extra credit if answered correctly.

Question 1.

Let f be the function given by $f(x, y, z, w) = x^2 - y^2 + y^3 + yz + z^2 + w^2$.

(a) **(6p)** Compute the partial derivatives and the Hessian matrix of f.
(b) **(6p)** Find all stationary points of f, and classify them as local max, local min or saddle points.
(c) **(6p)** Is f convex? Is it concave?

Question 2.

We consider the matrix A given by

$$A = \begin{pmatrix} a & 1 & 0 & 0 \\ 1 & a & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

(a) **(6p)** Compute the determinant and rank of A.
(b) **(6p)** Determine all values of a such that A is positive semidefinite.
(c) **(6p)** Find all eigenvalues of A.

Question 3.

Solve the following differential equations:

(a) **(6p)** $y'' - 7y' + 10y = 4e^t - 5$
(b) **(6p)** $ty' + (2 - t)y = e^{2t}$ (when $t > 0$)
(c) **(6p)** $3y^2te^{-t}y' + (y^3 - 1)e^{-t} = te^{-t}y^3$ (when $t > 0$)

Question 4.

We consider the following Kuhn-Tucker problem:

$$\min\ f(x,y,z,w) = x^2 + y^2 + z^2 + w^2 \text{ subject to } \begin{cases} xy + 1 \leq 0 \\ 2zw + 8 \leq 0 \end{cases}$$

(a) **(6p)** Write down the Kuhn-Tucker conditions. Show that there is a solution of these conditions with $(x, y, z, w) = (1, -1, 2, -2)$, and find the corresponding multipliers.

(b) **(6p)** Show that $(x, y, z, w) = (1, -1, 2, -2)$ solves the Kuhn-Tucker problem.

(c) **(6p)** Estimate the minimum value in the Kuhn-Tucker problem we obtain when we replace the second constraint with $2zw + 7.9 \leq 0$.

(d) **Extra credits (6p)** Find all solutions of the Kuhn-Tucker conditions in a).