This exam has 8 questions

QUESTION 1.

Consider a linear system $A \cdot \mathbf{x} = \mathbf{b}$, where the coefficient matrix A is invertible. **Which statement is true?**

(a) The linear system has a unique solution
(b) The linear system is inconsistent
(c) The linear system has at least one degree of freedom
(d) The number of solutions of the linear system depends on $\mathbf{b}$.
(e) I prefer not to answer.

QUESTION 2.

Consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, given by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 3 \\ t \end{pmatrix}$$

Which statement is true?

(a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent for all t
(b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent exactly when $t = 6$
(c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 6$
(d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 7$
(e) I prefer not to answer.

QUESTION 3.

Consider the matrix

$$A = \begin{pmatrix} t & t & 1 \\ 1 & 1 & t \end{pmatrix}$$

Which statement is true?

(a) There is exactly one value of t such that $\mathrm{rk}(A) = 2$
(b) There are exactly two values of t such that $\mathrm{rk}(A) = 2$
(c) There are no values of t such that $\mathrm{rk}(A) = 2$
(d) There are infinitely many values of t such that $\mathrm{rk}(A) = 2$
(e) I prefer not to answer.

Consider the matrix

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Which statement is true?

    (a) A has three distinct eigenvalues
    (b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
    (c) A has an eigenvalue of multiplicity three
    (d) A has one eigenvalues of multiplicity one, and no other eigenvalues
    (e) I prefer not to answer.

Consider the matrix A given by

$$A = \begin{pmatrix} 1 & s & -1 \\ 0 & 0 & s \\ 0 & 0 & 1 \end{pmatrix}$$

Which statement is true?

    (a) A is diagonalizable exactly when $s = 1$ and $s = -1$
    (b) A is diagonalizable exactly when $s = 1$
    (c) A is diagonalizable exactly when $s = 0$
    (d) A is diagonalizable for all s
    (e) I prefer not to answer.

Consider the quadratic form

$$f(x, y, z) = x^2 - 2xy - 4xz - 4yz + 4z^2$$

Which statement is true?

    (a) f is positive semidefinite but not positive definite
    (b) f is positive definite
    (c) f is negative definite
    (d) f is indefinite
    (e) I prefer not to answer.

Consider the function $f(x, y, z) = x^4 + 2x^2 + 4xz + y^2 + z^4 + 2z^2$. **Which statement is true?**

    (a) f is concave and has a global maximum
    (b) f is convex and has a global minimum
    (c) f is neither convex nor concave but has a global minimum
    (d) f has no global minimum
    (e) I prefer not to answer.

Two firms compete in a market. Initially, Firm A has 60% of the market, leaving Firm B with 40%. We model quarter-to-quarter changes in market shares as a Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$, where

$$\mathbf{x}_{t+1} = \begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} 0.64 & 0.18 \\ 0.36 & 0.82 \end{pmatrix} \cdot \begin{pmatrix} x_t \\ y_t \end{pmatrix} = A \cdot \mathbf{x}_t$$

and where x_t and y_t are the market shares of Firm A and Firm B after t quarters. In the long run, the equilibrium market shares are given by

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \lim_{t \to \infty} \mathbf{x}_t = \lim_{t \to \infty} \begin{pmatrix} x_t \\ y_t \end{pmatrix}$$

Which statement is true?

(a) Equilibrium market share x of Firm A is at least 75%.
(b) Equilibrium market share x of Firm A is between 50% and 75%.
(c) Equilibrium market share x of Firm A is between 25% and 50%.
(d) Equilibrium market share x of Firm A is less than 25%.
(e) I prefer not to answer.