This exam has 8 questions

QUESTION 1.

Consider a homogeneous linear system $A \cdot \mathbf{x} = \mathbf{0}$, where the coefficient matrix A has a pivot position in every column. **Which statement is true?**

(a) The linear system has a unique solution
(b) The linear system is inconsistent
(c) The linear system has one degree of freedom
(d) The linear system has two degrees of freedom
(e) I prefer not to answer.

QUESTION 2.

Consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, given by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 4 \\ t \end{pmatrix}$$

Which statement is true?

(a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent for all t
(b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent exactly when $t = 6$
(c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 6$
(d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 7$
(e) I prefer not to answer.

QUESTION 3.

Consider the matrix

$$A = \begin{pmatrix} t & 1 & t \\ 1 & t & 1 \end{pmatrix}$$

Which statement is true?

(a) There is exactly one value of t such that $\text{rk}(A) = 2$
(b) There are exactly two values of t such that $\text{rk}(A) = 2$
(c) There are no values of t such that $\text{rk}(A) = 2$
(d) There are infinitely many values of t such that $\text{rk}(A) = 2$
(e) I prefer not to answer.

Consider the matrix

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Which statement is true?

(a) A has three distinct eigenvalues
(b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
(c) A has an eigenvalue of multiplicity three
(d) A has one eigenvalues of multiplicity one, and no other eigenvalues
(e) I prefer not to answer.

Consider the matrix A given by

$$A = \begin{pmatrix} 1 & s & -s^2 \\ 0 & 0 & s \\ 0 & 0 & 1 \end{pmatrix}$$

Which statement is true?

(a) A is diagonalizable for all s
(b) A is diagonalizable exactly when $s = 1$
(c) A is diagonalizable exactly when $s = 0$
(d) A is not diagonalizable for any s
(e) I prefer not to answer.

Consider the quadratic form

$$f(x, y, z) = -x^2 + 4xy + 2xz - 4y^2 + 4yz$$

Which statement is true?

(a) f is negative semidefinite but not negative definite
(b) f is negative definite
(c) f is positive definite
(d) f is indefinite
(e) I prefer not to answer.

Consider the function $f(x, y, z) = 1 - x^4 - 2x^2 - 4xz - y^2 - z^4 - 2z^2$. **Which statement is true?**

(a) f has a saddle point at $(x, y, z) = (0, 0, 0)$.
(b) f has a global maximum point at $(x, y, z) = (0, 0, 0)$.
(c) f has a global minimum point at $(x, y, z) = (0, 0, 0)$.
(d) f has a local maximum point at $(x, y, z) = (0, 0, 0)$, but no global maximum value.
(e) I prefer not to answer.

Two firms compete in a market. Initially, Firm A has 60% of the market, leaving Firm B with 40%. We model quarter-to-quarter changes in market shares as a Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$, where

$$\mathbf{x}_{t+1} = \begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} 0.77 & 0.17 \\ 0.23 & 0.83 \end{pmatrix} \cdot \begin{pmatrix} x_t \\ y_t \end{pmatrix} = A \cdot \mathbf{x}_t$$

and where x_t and y_t are the market shares of Firm A and Firm B after t quarters. In the long run, the equilibrium market shares are given by

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \lim_{t \to \infty} \mathbf{x}_t = \lim_{t \to \infty} \begin{pmatrix} x_t \\ y_t \end{pmatrix}$$

Which statement is true?

(a) Equilibrium market share x of Firm A is at least 75%.
(b) Equilibrium market share x of Firm A is between 50% and 75%.
(c) Equilibrium market share x of Firm A is between 25% and 50%.
(d) Equilibrium market share x of Firm A is less than 25%.
(e) I prefer not to answer.

SVARARK TIL FLERVALGSEKSAMEN
ANSWER SHEET FOR MULTIPLE CHOICE EXAMINATION

Eksamenskode:
Examination code:

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Initialer:
Personal initials:

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ID-nummer: (**SKAL** fylles ut!)
ID-number: (**MUST** be filled in!)

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Skriv tydelig!
Fyll ut med ☒
Annuler kryss med ■
Helt fylt rute blir ikke registrert

Write clearly!
Record answer with ☒
Cancel a cross with ■
Compl. filled boxes will not be registered

Dette svararket leses kun av en maskin. Ikke noe av det du skriver utenom de definerte feltene blir lest eller tatt hensyn til.
Ikke kluss på arket. Be heller om et nytt.

This answer sheet is only read by a machine. Answers or comments written on the examination paper or outside the boxes will not be graded.
Do not scribble on this sheet.
Please ask for a new answer sheet if you need one.

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