NORWEGIAN BUSINESS SCHOOL

Solutions:	**GRA 60352**	**Mathematics**		
Examination date:	09.10.2015	15:00 – 16:00	Total no. of pages:	2
			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Ordinary exam			Responsible department: Economics	

Correct answers: D-C-D-C-A-D-B-C

Question 1.

The linear system is consistent with a three degrees of freedom since it has rank 2 (that is, a pivot in two of the first five columns). The correct answer is alternative **D**.

Question 2.

We form the matrix with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its determinant

$$\begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 1 \\ s & 1 & 3 \end{vmatrix} = 1(9-1) + 1(6-4) + s(2-12) = 10 - 10s$$

This shows that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent when $s \neq 1$, and linearly dependent if $s = 1$. The correct answer is alternative **C**.

Question 3.

We use elementary row operations to find an echelon form:

$$\begin{vmatrix} 1 & 4 & -7 & 3 \\ 3 & 2 & 1 & 3 \\ 4 & 6 & t & 1-t \end{vmatrix} \to \begin{vmatrix} 1 & 4 & -7 & 3 \\ 0 & -10 & 22 & -6 \\ 0 & -10 & t+28 & -t-11 \end{vmatrix} \to \begin{vmatrix} 1 & 4 & -7 & 3 \\ 0 & -10 & 22 & -6 \\ 0 & 0 & t+6 & -t-5 \end{vmatrix}$$

It follows that the rank of A is 3 for all values of t since the last row will have a pivot for all values of t. The correct answer is alternative **D**.

Question 4.

The characteristic equation of A is

$$\begin{vmatrix} 1-\lambda & \sqrt{2} & 0 \\ \sqrt{3} & 1-\lambda & 0 \\ 0 & 0 & -6-\lambda \end{vmatrix} = (-6-\lambda)(\lambda^2 - 2\lambda + 1 - \sqrt{6}) = 0$$

Hence the eigenvalues of A are $\lambda_1 = -6$ and λ_2, λ_3 such that $\lambda_2 + \lambda_3 = 2,\ \lambda_2\lambda_3 = 1 - \sqrt{6}$. Since $1 - \sqrt{6} < 0$, exactly one of the eigenvalues λ_2, λ_3 are negative. The correct answer is alternative **C**.

Question 5.

The eigenvalues are the numbers $1, 2, 3$ on the diagonal since A is upper triangular. The matrix is diagonalizable since it has three distinct eigenvalues. The correct answer is alternative **A**.

Question 6.

Eigenvectors for $\lambda = 1$ are given by the linear system $(A - I)\mathbf{x} = \mathbf{0}$, where

$$A - I = \begin{pmatrix} -0.26 & 0.13 \\ 0.26 & -0.13 \end{pmatrix}$$

Therefore, we see that $x = 1$ and $y = 2$ gives one eigenvector, and all others are multiple of this one. Multiplication by $1/3$ gives the state vector with $x = 1/3$ and $y = 2/3$. The correct answer is alternative **D**.

Question 7.

The symmetric matrix of the quadratic form $f(x_1, x_2, x_3, x_4) = 2x_1^2 + 6x_1x_2 + 5x_2^2 - 2x_2x_3 + 3x_3^2 + 2x_3x_4 + 4x_4^2$ is given by

$$A = \begin{pmatrix} 2 & 3 & 0 & 0 \\ 3 & 5 & -1 & 0 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

The leading principal minors are $D_1 = 2$, $D_2 = 10 - 9 = 1$, $D_3 = 3D_2 + 1(-2) = 1$, $D_4 = 4D_3 - 1 = 3$. Since all leading principal minors are positive, f is positive definite. The correct answer is alternative **B**.

Question 8.

The function $f(x, y, z) = x^a\sqrt{y} = x^a y^{0.5}$ has Hessian matrix

$$H(f) = \begin{pmatrix} a(a-1)x^{a-2}y^{0.5} & 0.5ax^{a-1}y^{-0.5} \\ 0.5ax^{a-1}y^{-0.5} & -0.25x^a y^{-1.5} \end{pmatrix}$$

Hence $D_2 = 0.25x^{2a-2}y^{-1} \cdot (-a(a-1) - a^2) = 0.25a(1-2a)x^{2a-2}y^{-1}$, so $D_2 \geq 0$ when $a \leq 1/2$. In this case, all first order principal minors are negative, so f is concave. If $a > 1/2$, then $D_2 < 0$ and f is neither convex nor concave. The correct answer is alternative **C**.