BI NORWEGIAN BUSINESS SCHOOL

Solutions:	**GRA 60352**	**Mathematics**		
Examination date:	27.04.2015	09:00 – 10:00	Total no. of pages:	3
			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Re-take exam			Responsible department: Economics	

Correct answers: A-C-B-D-C-C-B-D

Question 1.

The linear system is consistent with a unique solution since it has rank 3 (that is, one pivot in each of the first three columns). The correct answer is alternative **A**.

Question 2.

We form the matrix with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its determinant

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & a-1 \\ 1 & a & 5 \end{vmatrix} = 1(10 - a(a-1)) - 1(5-2a) + 1(a-1-4) = -a^2 + 4a = a(4-a)$$

This shows that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent when $a \neq 0$ and $a \neq 4$, and linearly dependent if $a = 0$ or $a = 4$. The correct answer is alternative **C**.

Question 3.

We compute the minor of order 3 obtained from the first three columns:

$$\begin{vmatrix} h & 4 & 7 \\ 3 & 1 & 0 \\ 2 & 4 & 7 \end{vmatrix} = 7 \cdot 10 + 7(h-12) = 7(h-2)$$

It follows that the rank of A is 3 if $h \neq 2$. If $h = 2$, we see that the first row is equal to the last rows, so the rank is at most two. Since column two and three are linearly independent, the rank is two. The correct answer is alternative **B**.

Question 4.

The characteristic equation of A is

$$\begin{vmatrix} 1-\lambda & 0 & -3 \\ 0 & -1-\lambda & 0 \\ 4 & 0 & -6-\lambda \end{vmatrix} = (-1-\lambda)(\lambda^2+5\lambda+6) = 0$$

Hence the eigenvalues of A are $\lambda = -1$ and $\lambda = -5/2 \pm 1/2 = -3, -2$. The correct answer is alternative **D**.

Question 5.

The eigenvalues are the numbers $1, 1, s$ on the diagonal since A is lower triangular. When $s \neq 1$, then the eigenvalues are $\lambda = 1$ (with multiplicity two) and $\lambda = s$ (with multiplicity one). For $\lambda = 1$, $\text{rk}(A - I) = 1$ and the linear system has two free variables. Therefore, the matrix is diagonalizable when $s \neq 1$. If $s = 1$, then the only eigenvalue is $\lambda = 1$ (with multiplicity three). For $\lambda = 1$, $\text{rk}(A - I) = 1$ so the linear system has two degrees of freedom, and the matrix is not diagonalizable. The correct answer is alternative **C**.

Question 6.

The symmetric matrix of the quadratic form $f(x_1, x_2, x_3, x_4) = -x_1^2 + 4x_1x_2 + 3x_1x_3 - 5x_2^2 - 6x_3^2 - x_4^2$ is given by

$$A = \begin{pmatrix} -1 & 2 & 3/2 & 0 \\ 2 & -5 & 0 & 0 \\ 3/2 & 0 & -6 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The leading principal minors are $D_1 = -1$, $D_2 = 5 - 4 = 1$, $D_3 = -6D_2 + 3/2(0 - 3/2(-5)) = -6 + 45/4 = 21/4$. Since D_1 and D_3 have opposite signs, f is indefinite. The correct answer is alternative **C**.

Question 7.

We compute the first order derivatives to find stationary points, and find

$$4x^3 + 4y = 0, \quad 4x + 4y^3 = 0$$

This gives $y = -x^3$ and $x = -y^3 = -(-x^3)^3 = x^9$, or $x(1 - x^8) = 0$. There are three stationary points $(0, 0), (1, -1), (-1, 1)$ corresponding to the solutions $x = 0$, $x = 1$ and $x = -1$. We compute the Hessian matrix of f and find

$$H(f) = \begin{pmatrix} 12x^2 & 4 \\ 4 & 12y^2 \end{pmatrix}$$

At $(0, 0)$ the matrix is indefinite since $D_2 = -16 < 0$. At the two other points, $D_1 = 12 > 0$ and $D_2 = 144 - 16 > 0$ so the matrix is positive definite. Hence $(1, -1)$ and $(-1, 1)$ are local minima while $(0, 0)$ is a saddle point. The correct answer is alternative **B**.

Question 8.

The function $f(x, y, z) = x^4 + hxy + y^4 + z^4 + z^2$ has Hessian matrix

$$H(f) = \begin{pmatrix} 12x^2 & h & 0 \\ h & 12y^2 & 0 \\ 0 & 0 & 12z^2 + 2 \end{pmatrix}$$

Hence $D_2 = 144x^2y^2 - h^2$ takes both positive and negative values if $h \neq 0$, so f is neither convex nor concave when $h \neq 0$. If $h = 0$, then the Hessian is a diagonal matrix with diagonal entries (that is, eigenvalues) $12x^2, 12y^2, 12z^2 + 2 \geq 0$. Hence f is convex when $h = 0$. The correct answer is alternative **D**.