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**Multiple-choice examination in: GRA 60352 Mathematics**

Examination date: 24.09.2010, 14:00 – 15:00

Permitted examination aids: Bilingual dictionary.

BI-approved exam calculator: TEXAS INSTRUMENTS BA II Plus™

Answer sheets: Answer sheet for multiple choice examinations

Total number of pages: 4

Number of attachments: 1 (example of how to use the answer sheet)

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**PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!**

- Students must themselves assure that the examination papers are complete.
- Students must provide the following information on the answer sheet:
  - Examination code
  - Personal initials
  - ID number

The student registration number must be recorded with both the appropriate numbers and by putting an “X” by the corresponding number in the columns below.

- Pens with green ink and pencils cannot be used in filling in answer sheets. Answer sheets must not be used for writing rough drafts.
- **All answers must be recorded with an “X” under the letter you believe corresponds with the correct answer.**
- **Cancel an “X” by filling in the box completely (boxes that are completely filled in will not be registered). “X” in two boxes for one question will be registered as a wrong answer.**
- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An “X” under E indicates that you choose not to answer question 5.
- **Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.**
- There is only one right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the simplest questions first.
- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an “X” next to E”) and correct answers are given 3 points.
- You can keep the examination papers.

Good luck!

## This exam has 8 questions

### QUESTION 1.

Consider the linear system

$$\begin{pmatrix} 1 & -3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \\ 0 \end{pmatrix}$$

Which statement is true?

- (A) The linear system is inconsistent.
- (B) The linear system has a unique solution.
- (C) The linear system has one degree of freedom
- (D) The linear system has two degrees of freedom
- (E) I prefer not to answer.

### QUESTION 2.

Consider the set of vectors  $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ , where

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ h \end{pmatrix}$$

and  $h$  is a parameter. Which statement is true?

- (A)  $\mathcal{B}$  is a linearly independent set of vectors for all  $h$
- (B)  $\mathcal{B}$  is a linearly independent set of vectors exactly when  $h = 3$
- (C)  $\mathcal{B}$  is a linearly independent set of vectors exactly when  $h \neq 1/7$
- (D)  $\mathcal{B}$  is a linearly independent set of vectors exactly when  $h \neq 3$
- (E) I prefer not to answer.

### QUESTION 3.

Compute the rank of the matrix

$$A = \begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & -2 & 4 \end{pmatrix}$$

Which statement is true?

- (A)  $\text{rk } A = 1$
- (B)  $\text{rk } A = 2$
- (C)  $\text{rk } A = 3$
- (D)  $\text{rk } A = 4$
- (E) I prefer not to answer.

QUESTION 4.

Consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$$

Which statement is true?

- (A)  $A$  has eigenvalues  $\lambda = 1$  and  $\lambda = -1$
- (B)  $A$  has eigenvalues  $\lambda = 2$  and  $\lambda = 4$
- (C)  $A$  has a single eigenvalue  $\lambda = 3$
- (D)  $A$  has eigenvalues  $\lambda = 3$  and  $\lambda = -3$
- (E) I prefer not to answer.

QUESTION 5.

Consider the matrix

$$A = \begin{pmatrix} 5 & 4 \\ -1 & 9 \end{pmatrix}$$

Which statement is true?

- (A)  $\lambda = 7$  is not an eigenvalue for  $A$
- (B)  $\lambda = 7$  is an eigenvalue for  $A$  and  $A$  is diagonalizable
- (C)  $\lambda = 7$  is an eigenvalue for  $A$  but  $A$  is not diagonalizable
- (D)  $\lambda = 7$  and  $\lambda = 3$  are eigenvalues for  $A$
- (E) I prefer not to answer.

QUESTION 6.

Consider the quadratic form

$$Q(x_1, x_2) = x_1^2 - 4x_1x_2 + 4x_2^2$$

Which statement is true?

- (A)  $Q$  is positive semidefinite but not positive definite
- (B)  $Q$  is negative semidefinite but not negative definite
- (C)  $Q$  is indefinite
- (D)  $Q$  is positive definite
- (E) I prefer not to answer.

QUESTION 7.

Consider the function

$$f(x_1, x_2, x_3) = 3x_1^2 + 2x_1x_2 + 3x_2^2 + x_3^2 + x_1 - x_2$$

defined on  $\mathbb{R}^3$ . Which statement is true?

- (A)  $f$  is a convex function but not a concave function
- (B)  $f$  is a convex function and a concave function
- (C)  $f$  is not a convex function but a concave function
- (D)  $f$  is neither a convex nor a concave function
- (E) I prefer not to answer.

QUESTION 8.

A car rental agency has two rental locations in a certain city, one downtown and one at the airport. Let  $x_t$  denote the number of cars at the downtown location after  $t$  days, and  $y_t$  the number of cars at the airport after  $t$  days. We assume that the distribution of cars is given by the following model:

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = A \cdot \begin{pmatrix} x_t \\ y_t \end{pmatrix}, \quad \text{where } A = \begin{pmatrix} 0.97 & 0.02 \\ 0.03 & 0.98 \end{pmatrix}$$

A steady state for this system is a state  $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$  such that  $A\mathbf{v} = \mathbf{v}$ . **Which statement is true?**

- (A) The system has a steady state with 30% of the cars at the downtown location
- (B) The system has a steady state with 40% of the cars at the downtown location
- (C) The system has a steady state with 50% of the cars at the downtown location
- (D) The system has a steady state with 60% of the cars at the downtown location
- (E) I prefer not to answer.