EXAMINATION QUESTION PAPER - Written examination

GRA 60353
Mathematics

Department of Economics

Start date:	20.06.2018	Time 09.00
Finish date:	20.06.2018	Time 12.00

Weight:	80% of GRA 6035
Total no. of pages:	2 incl. front page
Answer sheets:	Squares
Examination support materials permitted:	BI-approved exam calculator. Simple calculator. Bilingual dictionary.

Re-sit	Ordinary

Exam	Final exam in GRA 6035 Mathematics
Date	June 20th, 2018 at 0900 - 1200

This exam consists of 12+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 72p (12 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.

We consider the matrix A and the vector $\mathbf{b}$ given by

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & t & t^2 \\ 1 & -t & t^2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ t^3 \\ -t^3 \end{pmatrix}$$

(a) **(6p)** Compute the determinant of A.
(b) **(6p)** Compute the rank of A.
(c) **(6p)** Determine the values of t such that the linear system $A\mathbf{x} = \mathbf{b}$ is consistent.

Question 2.

Find the general solutions of the following differential equations:

(a) **(6p)** $y'' - 29y' + 100y = 100t - 29$
(b) **(6p)** $y' + 2ty = 4e^{-t^2}$
(c) **(6p)** $ty' = y\ln(y)$

Question 3.

Let u be the function given by $u(x, y, z) = 2x^2 - 2xy - 4xz + y^2 + 4z^2 - 2$, and consider the composite function $f(x, y, z) = e^u + e^{-u}$ with $u = u(x, y, z)$.

(a) **(6p)** Find all stationary points of $u = u(x, y, z)$.
(b) **(6p)** Determine the minimal value of $u = u(x, y, z)$, if it exists.
(c) **(6p)** Determine the maximum and minimum values of f, it they exist.

Question 4.

We consider the following Kuhn-Tucker problem:

$$\max\ f(x, y) = xy(x - y) \text{ subject to } x^2 + y^2 + (x - y)^2 \le 96$$

(a) **(6p)** Write down all Kuhn-Tucker conditions for this problem.
(b) **(6p)** Find all points $(x, y; \lambda)$ with $x, y \neq 0$ that satisfy the Kuhn-Tucker conditions.
(c) **(6p)** Show that the Kuhn-Tucker problem has a maximum, and find the maximum value.

Question 5.

Extra credit (6p)
Solve the logistic differential equation $y' = ry(1 - y/K)$ when $r > 0$ and $K > 0$ are given constants. Sketch the solution curve $y = y(t)$, showing the equilibrium states and their stability properties.