Exam	Final exam in GRA 6035 Mathematics
Date	December 1st, 2016 at 0900 - 1200

All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 5 can be skipped, but gives 6p extra credit if answered correctly.

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} -2 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & -1 & 2 \end{pmatrix}$$

(a) **(6p)** Compute the determinant and rank of A.
(b) **(6p)** Solve the linear system $A \cdot \mathbf{x} = \mathbf{0}$, and write the solutions in the form $\text{span}(\mathbf{v}_1, \ldots, \mathbf{v}_r)$.
(c) **(6p)** Find all eigenvalues of A and their multiplicities.

Question 2.

Solve the difference equation:

(a) **(6p)** $y_{t+2} = 3y_{t+1} - 2y_t,\ y_0 = 1,\ y_1 = 2$

Solve the differential equations:

(b) **(6p)** $y' - y \ln t = y$
(c) **(6p)** $ye^{ty} + te^{ty}y' = 1,\ y(1) = \ln 2$

Question 3.

We consider the function given by $f(x, y, z) = 5x^2 - 8xy - 4xz + 5y^2 - 4yz + 8z^2 + 1$.

(a) **(6p)** Is f convex?
(b) **(6p)** Find all the stationary points of f.
(c) **(6p)** Find the minimum value of $g(x, y, z) = w \ln(w)$, with $w = f(x, y, z)$, if it exists.

Question 4.

We consider the following Lagrange problem:

$$\min\ f(x, y, z) = 5x^2 - 8xy - 4xz + 5y^2 - 4yz + 8z^2 + 1 \text{ subject to } x + y - 4z = 8$$

(a) **(6p)** Write the Lagrange conditions as a linear system and find its augmented matrix.
(b) **(6p)** Solve the Lagrange problem. What is the minimum value?
(c) **(6p)** Consider the new Lagrange problem where the constraint is replaced by $x+y-4z = 7.92$. State the relevant envelope theorem, and use it to estimate the new minimum value.

Question 5.

Let $\alpha_1, \alpha_2, \alpha_3$ be parameters and consider the matrix A given by

$$A = \begin{pmatrix} -\alpha_2 & \alpha_1 & 0 \\ -\alpha_3 & 0 & \alpha_1 \\ 0 & -\alpha_3 & \alpha_2 \end{pmatrix}$$

Extra credits (6p)
Compute the rank of A for all values of $(\alpha_1, \alpha_2, \alpha_3)$.