Exam	Final exam in GRA 6035 Mathematics
Date	May 15th, 2015 at 0900 - 1200

All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 4c can be skipped, but gives 6p extra credit if answered correctly.

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} s & s & s \\ 0 & s & s \\ 0 & 0 & s \end{pmatrix}$$

(a) **(6p)** Compute the determinant and rank of A.
(b) **(6p)** Compute all eigenvalues and eigenvectors of A when $s = 1$.
(c) **(6p)** For which values of s is A diagonalizable?

Three companies A, B and C share the market for a certain commodity. Initially, company A has 50% of the marked, company B has 35% and company C has 15%. We represent the market shares of the three companies after t years as a marked share vector $\mathbf{x}_t$, and assume that the changes in marked shares are given by a Markov chain $\mathbf{x}_{t+1} = T\mathbf{x}_t$, with

$$T = \begin{pmatrix} 0.70 & 0.30 & 0.50 \\ 0.20 & 0.50 & 0.20 \\ 0.10 & 0.20 & 0.30 \end{pmatrix}, \quad \mathbf{x}_t = \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix}$$

where T is the transition matrix and x_t, y_t, z_t represents the marked shares of company A,B,C after t years.

(d) **(6p)** Find the market shares of all three companies in the long run.

Question 2.

Find the general solution of the differential equations:

(a) **(6p)** $y'' - 3y' - 10y = t$
(b) **(6p)** $t^2y' + ty = \ln t$
(c) **(6p)** $(\ln t - 6ty)y' = 3y^2 - y/t$

Question 3.

We consider the function given by $f(x, y) = \ln(x^2 + y^2 + 1) - x^2 - y^2$.

(a) **(6p)** Compute the partial derivatives of the function f, and find its stationary points.
(b) **(6p)** Compute the Hessian matrix of f at $(x, y) = (1, 0)$, and determine its definiteness.
(c) **(6p)** Can you conclude that f is concave based on your answer in Question (b)?

Question 4.

We consider the following Kuhn-Tucker problem:

$$\max\ f(x,y) = 2\ln(x^2+y^2+1) - x^2 - y^2 \text{ subject to } 2xy \geq 1$$

(a) **(6p)** Write down the Kuhn-Tucker conditions for this problem, and find all points $(x, y; \lambda)$ that satisfy these conditions and have both $\lambda = 0$ and binding constraint.

(b) **(6p)** Is the set of admissible points bounded? Are there admissible points where NDCQ fails?

(c) **Extra credits (6p)** Solve the Kuhn-Tucker problem and find the corresponding maximum value.