

**Written examination: GRA 60353 Mathematics**

Examination date:	06.06.2012	09:00 – 12:00	Total no. of pages: 1
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus		
Answer sheets:	Squares		
	Counts 80% of GRA 6035	The subquestions are weighted equally	
Re-sit exam	Responsible department: Economics		

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QUESTION 1.

We consider the function  $f$  given by  $f(x, y, z) = 7xy + 5y^2 - (z - x)^4$ .

- (a) Find all the stationary points of  $f$ .
- (b) Is  $f$  convex? Is it concave?

QUESTION 2.

Find the general solution  $y = y(t)$  of the following differential equations:

- (a)  $y'' - 7y' + 12y = t - 3$
- (b)  $1 - 3y^2y' = te^t$
- (c)  $(t/y) \cdot y' + \ln y = 1$

QUESTION 3.

We consider the matrix  $A$  and the vector  $\mathbf{b}$  given by

$$A = \begin{pmatrix} 5 & -5 & 15t - 35 \\ 2 & t - 4 & 7t - 16 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2t \\ t \end{pmatrix}$$

- (a) What is the rank of  $A$ ? Are the column vectors of  $A$  linearly independent for any values of  $t$ ?
- (b) For which values of  $t$  does the linear system  $A\mathbf{x} = \mathbf{b}$  have one solution, infinitely many solutions and no solutions?
- (c) How many degrees of freedom does the linear system  $(A^T A)\mathbf{x} = \mathbf{0}$  have? (It is not necessary to compute  $A^T A$  to answer this question.)

QUESTION 4.

We consider the optimization problem

$$\max x^2yz \text{ subject to } x^2 + 2y^2 - 2z^2 \leq 32$$

- (a) Write down the first order conditions and the complementary slackness conditions for the maximum problem, and find all admissible points that satisfy these conditions.
- (b) Does the maximum problem have a solution?